SO5032 Quantitative Research Methods

Brendan Halpin, Sociology, University of Limerick

Spring 2024

Outline

Lecture 0: Course Outline

Lecture 1: Categorical data analysis

Lecture 2: Ordinal association

Lecture 3: Multidimensional causality

Lecture 4: Summary of multiple regression

Lecture 5: Interaction and Non-linearity

Lecture 6: Residuals and Influence

Lecture 7: Logs and log regression

Lecture 9: Logistic regression

Lecture 10: Logistic regression continued

Lecture 11: Multinomial and Ordinal regression

SO5032 Spring 2023/4 – Module outline

Module Code:	SO5032
Module Title:	Quantitative Research Methods II (MA)
Academic Year:	2023/4
Semester:	Spring
Lecturer(s):	Dr Brendan Halpin
Lecture Locations:	Lec Mon 09-1100 P1006, Lab Weds 12-1400 A0060a
Lecturer(s) Contact Details:	brendan.halpin@ul.ie
Lecturer(s) Office Hours:	Mon 1100-1300

Short Summary of Module:

Intermediate quantitative research methods for sociology, following on from SO5041.

Aims and Objectives of Module:

- A continuation of SO5041 – builds on what was learnt there
- A deeper look at methods already covered, especially regression
- Related methods more suited to social science data: methods for categorical and ordinal variables, including logistic regression
- Further use of Stata:
  - Use in a production environment – do-files, logging, reproducibility
  - More complex data handling
  - Further analytic procedures
- Secondary analysis: real research with existing data sets

Learning Outcomes:

- Deeper understanding of methods for analysis of categorical data
- Understanding of the nature of multivariate causality
- Understanding of the theory and practice of multiple linear regression
- An understanding of some methods for regression with categorical dependent variables
- Deeper understanding of sampling practice and theory
- Practical skills for accessing and analysing large-scale data sets
- An ability to read quantitative social research
- Greater competence in Stata, particularly for handling larger projects

Course Structure:

One two-hour lecture per week, one two-hour lab per week.

Detailed outline

- Revisit χ^2, look at methods for more complex analysis of categorical (nominal *and* ordinal) data (chapter 8, Agresti)(1-2 weeks)
- Multivariate causality (chapter 10 from Agresti) (1 week)
- Multiple regression (chapters 11, 14 from Agresti) (3 weeks plus)
- More sampling theory: clusters, strata, weighting (1 week)
- Data sets, data archives and secondary analysis (1 week, ongoing in labs)
- Logistic regression: regression where the dependent variable is binary (or multinomial) rather than continuous (chapter 15 from Agresti) (3 weeks plus)
- Reading statistical research – what gets published and how to read it (1-2 weeks/on-going)

Lecture topics by week

Week beginning	Topic	Lecture Mon 09-1100	Lab Wed 12-1400
1: Jan 29	Categorical data, association in tables	✓	✓
2: Feb 05	Association in ordinal data	*X*	✓ (lecture)
3: Feb 12	Understanding multidimensional causality	✓	✓
4: Feb 19	Introducing multiple regression	✓	✓
5: Feb 26	Further multiple regression	✓	✓
6: Mar 04	Multiple regression: residuals & influence	✓	✓
7: Mar 11	Regression with logged dependent variables	✓	✓
8: Mar 18	Introducing logistic regression	*X*	✓ (lecture)
9: Apr 01	Further logistic regression	*X*	✓ (lecture)
10: Apr 08	Multinomial regression	✓	✓
11: Apr 15	Multinomial and ordinal regression	✓	✓
12: Apr 22	Ordinal regression continued	✓	✓

Texts

- Main text: Agresti, *Statistical Methods for the Social Sciences* – particularly chapters 8, 10, 11, 14 and 15
- Supplementary texts:
  - de Vaus, *Surveys in Social Research*: good on survey methodology
  - Agresti, *Introduction to Categorical Data Analysis*
  - Pevalin and Robson, *The Stata Survival Manual*

Details of Module Assessment:

- Three assignments, weeks 6, 11 and 15.
- The first two assignments are worth 20% each.
- The final assignment is a project, worth 60%, and should be worked on throughout the semester (see below).

Details of Annual Repeats:

A 100% assignment, to be submitted in the examination period.

BrightSpace and Other Classroom Technologies:

- The module will use BrightSpace for submission of assignments and for provision of materials.
- http://teaching.sociology.ul.ie/so5032 will also be used

IN TERM ASSIGNMENT(S):

- Assignment 1: Homework exercises relating to linear regression.
  - Marks: 20%
  - Deadline: End week 6
- Assignment 2: Homework exercises relating to categorical data analysis.
  - Marks: 20%
  - Deadline: End week 11
- Assignment 3: A project This will involve the use of large-scale survey data, and require the formulation of a research question, and its addressing using statistical analysis.
  - Marks: 60%
  - Deadline: End week 15.

FEEDBACK:

Detailed feedback on assignments 1 and 2 will be given in weeks 8 and 13, by e-mail and on request face-to-face. Feedback on assignment 3 will be provided on request after the semester.

Plagiarism notice

It hardly needs to be said, but all work must be your own. All material drawn from other sources must be clearly attributed. Passing off others' work as your own is considered academic dishonesty, and can be subject to substantial penalties. Please familiarise yourself with the departmental policy on plagiarism and use the coversheet declaration with all assignments (both available at http://www.ul.ie/sociology/ under Student Resources).

Deadline policy

Please also note the Department's policy on deadlines, also available at http://www.ul.ie/sociology/ under Student Resources.

Association between categorical variables

- Association between categorical variables: departure from independence
- Visible in patterns of percentages
- Three main questions (cf Agresti/Finlay p265)
  - Is there evidence of association?
  - What is the form of the association?
  - How strong is the association?

The χ^2 test

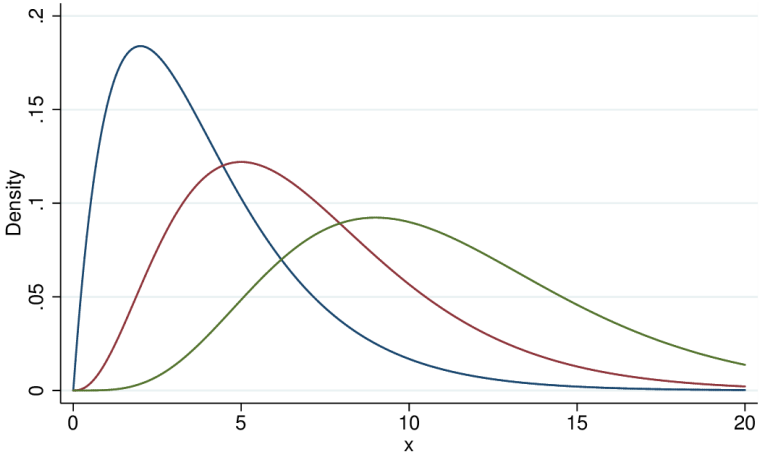
- Compare observed values with expected values under independence:

$$E = \frac{RC}{T}$$

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

- For frequency data, and for large samples the χ^2 statistic has a χ^2 distribution with $df = (r - 1)(c - 1)$
- Interpretation: chance of getting a χ^2 this big or bigger if H_0 (independence) is true in the population

The χ^2 distribution



Limitations of χ^2

- Large sample required: most expected counts 5+
- For frequency or count data, not rates or percentages
- Tests for *evidence* of association, not strength (see Agresti/Finlay Table 8.14, p 268)
- Looks for unpatterned association, may miss weak systematic association between ordinal variables

Pattern of association

- The form association takes is interesting
- We can see it by examining percentages
- Or residuals: $O - E$
- But residuals depend on sample and expected value size

Pearson residuals

- “Pearson residuals” are better:

$$\frac{O - E}{\sqrt{E}}$$

- Square and sum these residuals to get the χ^2 statistic

Adjusted Residuals

- The sum of squared Pearson residuals has a χ^2 distribution, but individually they are not normally distributed
- Adjusted residuals scale to have a standard normal distribution if independence holds:

$$AdjRes = \frac{O - E}{\sqrt{E(1 - \pi_r)(1 - \pi_c)}}$$

- Adjusted residuals outside the range -2 to +2 indicate cells with unusual observed values ($< c5\%$ chance)
- Adjusted residuals outside the range -3 to +3 indicate cells with very unusual observed values

Measures of association

- Evidence, pattern, now strength of association
- A number of measures
  - Difference of proportions
  - Odds ratio
  - Risk ratio (ratio of proportions)
- Focus on 2 by 2 pairs, but can be extended to bigger tables

Difference of proportions

No association

	Favour	Oppose	Total
White	360	240	600
Black	240	160	400
Total	600	400	1000

Maximal association

	Favour	Oppose	Total
White	600	0	600
Black	0	400	400
Total	600	400	1000

Difference in proportions

- Difference in proportions (i): $\frac{360}{600} - \frac{240}{400} = 0.6 - 0.6 = 0$
- Difference in proportions (ii): $\frac{600}{600} - \frac{0}{400} = 1 - 0 = 1$
- Range: -1 through 0 (no association) to +1

Relative risk

- “Relative risk” of ratio or proportions is also popular
- The ratio of two percentages:

$$RR = \frac{n_{11}/n_{1+}}{n_{21}/n_{2+}}$$

where n_{1+} indicates the row-1 total *etc.*

- Range = 0 through 1 (no association) to ∞

Odds ratios

- Odds differ from proportions/percentages:
  - Percentage: $\pi_i = \frac{f_i}{Total}$
  - Odds: $O_i = \frac{f_i}{Total - f_i} = \frac{\pi_i}{1-\pi_i}$
- Odds ratios are the ratios of two odds:

$$OR = \frac{n_{11}/n_{12}}{n_{21}/n_{22}}$$

- Range: 0 though 1 (no association) to ∞

Odds ratios

- Odds ratio (i): $\frac{\frac{360}{240}}{\frac{240}{160}} = \frac{1.5}{1.5} = 1$
- Odds ratio (ii): $\frac{\frac{600}{0}}{\frac{0}{400}} = \frac{\infty}{0} = \infty$
- Range: 0 through 1 (no association) to $+\infty$

Comparing measures

- Difference of proportions is simple and clear
- Ratio of proportions/Relative Risk is also simple
- Odds ratio is less intuitive but turns out to be mathematically more tractable
- DP and RR less consistent across different base levels of “risk”

Ordinal Data

- χ^2 may miss ordinal association
- Symmetric ordinal measures based on concordant and discordant pairs: γ (gamma), Kendall's τ (tau).

Reading (for this and last week):

- Agresti, Chapter 8

Lecture 2

- Expected values, residuals, adjusted residuals in Stata
- Ordinal association
- Association in multi-way tables
- Multivariate causality

Tabular association in Stata

`tabchi` procedure allows access to

- Percentages
- Expected values
- Residuals
- Adjusted residuals

Ordinal association

- When variables are ordinal, association may be structured
- High values on X are associated with high values on Y, low with low
- Or vice versa for negative association
- Analogous to correlation
- Examine using percentages, adjusted residuals: ordered pattern

Example: row percentages

```
. tab lopfamo lopfaml, row
```

Key
frequency *row percentage*

co-habiting is alright	divorce better than unhappy marriage					
	strongly	agree	neithr ag	disagree	stronglyd	Total
strongly agree	2,381	1,228	304	38	19	3,970
	59.97	30.93	7.66	0.96	0.48	100.00
agree	1,462	4,159	687	103	15	6,426
	22.75	64.72	10.69	1.60	0.23	100.00
neithr agree, disagr	485	1,803	717	73	13	3,091
	15.69	58.33	23.20	2.36	0.42	100.00
disagree	156	647	252	143	15	1,213
	12.86	53.34	20.77	11.79	1.24	100.00
stronglydisagree	78	143	129	101	50	501
	15.57	28.54	25.75	20.16	9.98	100.00
Total	4,562	7,980	2,089	458	112	15,201
	30.01	52.50	13.74	3.01	0.74	100.00

Example: observed and expected values

```
. tabchi lopfamo lopfaml
```

observed frequency
expected frequency

co-habiting is alright	divorce better than unhappy marriage				
	strongly agree	agree	neithr agree, disagr	disagree	stronglydisagree
strongly agree	2381	1228	304	38	19
	1191.444	2084.113	545.578	119.614	29.251
agree	1462	4159	687	103	15
	1928.519	3373.428	883.094	193.613	47.346
neithr agree, disagr	485	1803	717	73	13
	927.646	1622.668	424.781	93.131	22.774
disagree	156	647	252	143	15
	364.036	636.783	166.697	36.547	8.937
stronglydisagree	78	143	129	101	50
	150.356	263.008	68.850	15.095	3.691

```
1 cell with expected frequency < 5
        Pearson chi2(16) =  4.2e+03   Pr = 0.000
 likelihood-ratio chi2(16) =  3.3e+03   Pr = 0.000
```

Example: adjusted residuals

```
. tabchi lopfamo lopfaml, adj noo
```

expected frequency
adjusted residual

co-habiting is alright	divorce better than unhappy marriage				
	strongly agree	agree	neithr agree, disagr	disagree	stronglydisagree
strongly agree	1191.444	2084.113	545.578	119.614	29.251
	47.925	-31.654	-12.956	-8.815	-2.213
agree	1928.519	3373.428	883.094	193.613	47.346
	-16.713	25.829	-9.351	-8.703	-6.210
neithr agree, disagr	927.646	1622.668	424.781	93.131	22.774
	-19.463	7.277	17.104	-2.373	-2.303
disagree	364.036	636.783	166.697	36.547	8.937
	-13.587	0.612	7.416	18.639	2.122
stronglydisagree	150.356	263.008	68.850	15.095	3.691
	-7.173	-10.918	7.937	22.831	24.601

```
1 cell with expected frequency < 5
          Pearson chi2(16) =  4.2e+03   Pr = 0.000
 likelihood-ratio chi2(16) =  3.3e+03   Pr = 0.000
```

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Measures of ordinal association

- Sometimes Pearson's Correlation is used
- Equivalent to scoring the categories linearly and calculating the conventional correlation

```
. corr lopfamo lopfaml
(obs=15,201)
```

	lopfamo	lopfaml
lopfamo	1.0000	
lopfaml	0.3831	1.0000

Non-linear correlation

- Assumption of equal intervals problematic (but often reasonably OK)
- Spearman's Rank Correlation is a better solution

```
. spearman lopfamo lopfaml

 Number of obs =   15201
Spearman´s rho =       0.3840

Test of H0: lopfamo and lopfaml are independent
    Prob > |t| =       0.0000
```

Truly ordinal measures

- The Gamma statistic (γ) is truly ordinal
- Counts “concordant” and “discordant” pairs

$$\gamma = \frac{C - D}{C + D}$$

- Range: -1, 0, 1
- Approximately normal for large samples

Gamma in practice

```
. tab lopfamo lopfaml, gamma
```

co-habiting is alright	divorce better than unhappy marriage					
	strongly	agree	neithr ag	disagree	stronglyd	Total
strongly agree	2,381	1,228	304	38	19	3,970
agree	1,462	4,159	687	103	15	6,426
neithr agree, disagr	485	1,803	717	73	13	3,091
disagree	156	647	252	143	15	1,213
stronglydisagree	78	143	129	101	50	501
Total	4,562	7,980	2,089	458	112	15,201

gamma = 0.4975 ASE = 0.009

Variants

- Gamma is symmetrical
- Kendall's tau (τ) is also symmetrical, similar logic
- Somer's d also uses $C + D$ but is asymmetrical: one variable affecting another (takes account of ties)

Multi-way tables

- How do we think in terms of multi-way tables – more than two dimensions?
- Often, in terms of whether the $A \times B$ relationship is constant across C

Scouting example

Scout	Delinquent		
	Yes	No	Total
Yes	36	364	400
No	60	340	400
Total	96	704	800

Scouting example

Low Church Attendance Scout	Delinquent		
	Yes	No	Total
Yes	10	40	50
No	40	160	200
Total	50	200	250

Medium Church Attendance Scout	Delinquent		
	Yes	No	Total
Yes	18	132	150
No	18	132	150
Total	36	264	800

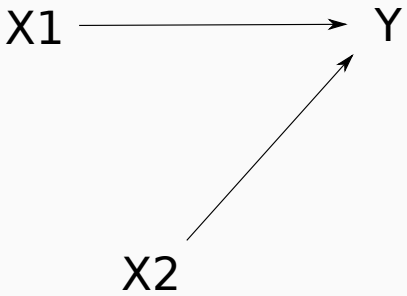
High Church Attendance Scout	Delinquent		
	Yes	No	Total
Yes	8	192	200
No	2	48	50
Total	10	240	250

Multidimensional causality

- Regression analysis never proves causal relationships, but it "thinks" in causal terms
- To use it we need to understand causal relationships: what process generates the data we see, and what can regression tell us about it.
- Start by considering the relationship between variables and patterns of association

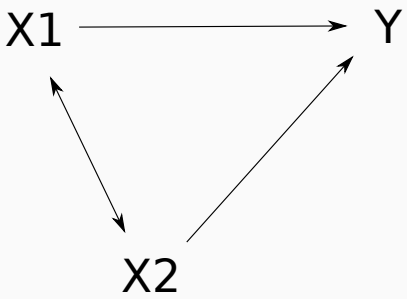
3-variable pictures

- Let's consider patterns of causality and association between three variables, X1 and X2, and Y
- If X1 and X2 are not correlated with each other, their separate effects on Y more or less just add up



Correlated X variables

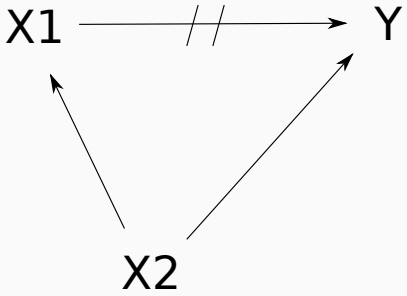
- But if X1 and X2 are correlated, things can get funny:



- In particular, if we measure the effect of one X without taking account of the other we will likely over-estimate it

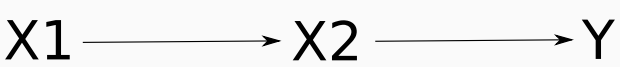
Spurious association

- X1 may have an association with Y, implying a causal relationship
- But if X2 affects both X1 and Y the relationship between X1 and Y may be spurious



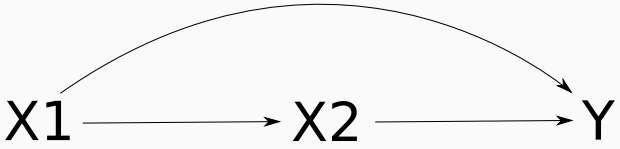
Indirect effects

- Where there is a time-order (X1 before X2), we may see direct and indirect effects
- X1 may affect X2, which affects Y, but not affect Y directly
- Thus there is association between X1 and Y without a direct causal effect



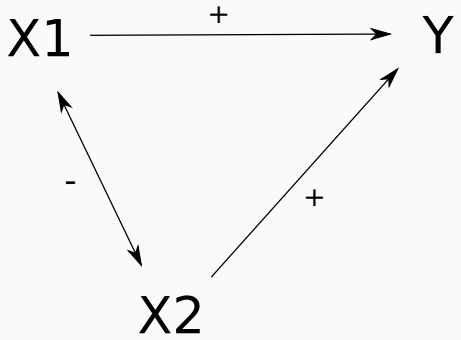
Direct and indirect effects

- However, it is possible for both direct and indirect effects to be present at the same time



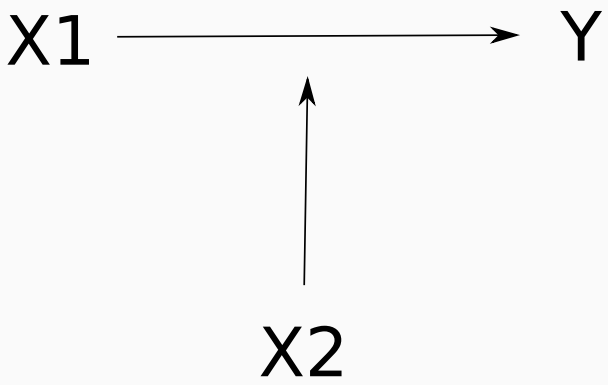
Suppression

- Where X1 and X2 have positive effects on Y, but a negative correlation, or different effects on Y with a positive correlation, the association between X1 and Y may be suppressed
- That is, it may be invisible if we don't take account of X2



Interactions

- An interaction effect is where the effect of one variable on Y changes depending on the value of another



Lecture 3: Multidimensional causality

Multiple regression

Multiple explanatory variables

- Regression analysis can be extended to the case where there is more than one explanatory variable – multivariate regression
- This allows us to estimate the net simultaneous effect of many variables, and thus to begin to disentangle more complex relationships
- Interpretation is relatively easy: each variable gets its own slope coefficient, standard error and significance
- The slope coefficient is the effect on the dependent variable of a 1 unit change in the explanatory variable, *while taking account of the other variables*

Example

- Example: income may be affected by gender, and also by paid work time: competing explanations – one or the other, or both could have effects
- We can fit bivariate regressions:

$$Income = a + b \times PaidWork$$

or

$$Income = a + b \times Female$$

- We can also fit a single multivariate regression

$$Income = a + b \times PaidWork + c \times Female$$

Dichotomous variables

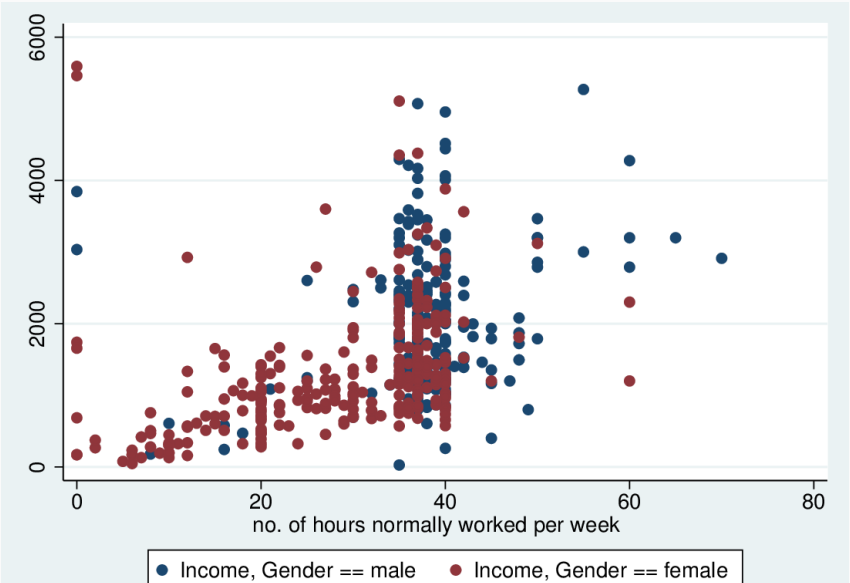
- We deal with gender in a special way: this is a *binary* or *dichotomous* variable – has two values
- We turn it into a yes/no or 0/1 variable – *e.g.*, female or not
- If we put this in as an explanatory variable a *one-unit change in the explanatory variable* is the difference between being male and female
- Thus the c coefficient we get in the $Income = a + b \times PaidWork + c \times Female$ regression is the net change in predicted income for females, once you take account of paid work time.
- The b coefficient is then the net effect of a unit change in paid work time, once you take gender into account.

Income, hours and gender

```
. corr Income Gender Hours
(obs=506)
```

	Income	Gender	Hours
Income	1.0000		
Gender	-0.3280	1.0000	
Hours	0.3638	-0.4360	1.0000

Income, hours and gender



T-test: Income by gender

```
. ttest Income, by(Gender)

Two-sample t test with equal variances
```

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
male	437	1618.348	59.11677	1235.809	1502.159	1734.537
female	531	992.1805	40.82127	940.6625	911.9892	1072.372
combined	968	1274.861	36.23219	1127.281	1203.759	1345.964
diff		626.1674	70.00484		488.7883	763.5465

```
    diff = mean(male) - mean(female)                              t =   8.9446
Ho: diff = 0                                     degrees of freedom =      966

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000
```

Regression: Just hours

```
. reg Income Hours
```

Source	SS	df	MS		
Model	86947928.8	1	86947928.8	Number of obs	= 506
Residual	570128215	504	1131206.78	F(1, 504)	= 76.86
				Prob > F	= 0.0000
Total	657076144	505	1301140.88	R-squared	= 0.1323
				Adj R-squared	= 0.1306
				Root MSE	= 1063.6

Income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Hours	37.82204	4.314061	8.77	0.000	29.34628	46.2978
_cons	449.7435	150.1722	2.99	0.003	154.703	744.7841

Regression: Hours and binary gender

```
. reg Income Hours i.Gender
```

Source	SS	df	MS		
Model	110236231	2	55118115.6	Number of obs	= 506
Residual	546839912	503	1087156.88	F(2, 503)	= 50.70
				Prob > F	= 0.0000
Total	657076144	505	1301140.88	R-squared	= 0.1678
				Adj R-squared	= 0.1645
				Root MSE	= 1042.7

Income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Hours	28.33857	4.699451	6.03	0.000	19.1056	37.57155
Gender						
female	-478.4214	103.3684	-4.63	0.000	-681.5084	-275.3344
_cons	1022.139	192.2717	5.32	0.000	644.3844	1399.893

Regression: for men only

```
. reg Income Hours if Gender==1
```

Source	SS	df	MS		
				Number of obs	= 232
				F(1, 230)	= 5.36
Model	8009519.02	1	8009519.02	Prob > F	= 0.0215
Residual	343845612	230	1494980.92	R-squared	= 0.0228
				Adj R-squared	= 0.0185
Total	351855131	231	1523182.38	Root MSE	= 1222.7

Income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Hours	24.61855	10.63597	2.31	0.022	3.662162	45.57495
_cons	1164.366	414.4901	2.81	0.005	347.6826	1981.049

Regression: for women only

```
. reg Income Hours if Gender==2
```

Source	SS	df	MS		
				Number of obs	= 274
				F(1, 272)	= 42.63
Model	31772944.2	1	31772944.2	Prob > F	= 0.0000
Residual	202744304	272	745383.469	R-squared	= 0.1355
				Adj R-squared	= 0.1323
Total	234517248	273	859037.537	Root MSE	= 863.36

Income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Hours	29.70376	4.549594	6.53	0.000	20.74687	38.66065
_cons	504.6153	140.3614	3.60	0.000	228.2824	780.9482

Regression: interaction

```
. reg Income c.Hours##i.Gender
```

Source	SS	df	MS	Number of obs	=	506
				F(3, 502)	=	33.82
Model	110486228	3	36828742.8	Prob > F	=	0.0000
Residual	546589915	502	1088824.53	R-squared	=	0.1681
				Adj R-squared	=	0.1632
Total	657076144	505	1301140.88	Root MSE	=	1043.5

Income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Hours	24.61855	9.076915	2.71	0.007	6.785132	42.45198
Gender						
female	-659.7502	392.3082	-1.68	0.093	-1430.518	111.0181
Gender#c.Hours						
female	5.085207	10.61255	0.48	0.632	-15.76529	25.9357
_cons	1164.366	353.7327	3.29	0.001	469.3865	1859.345

Regression: Direct and indirect 1

```
. reg ownscore fatherscore
```

Source	SS	df	MS		
				Number of obs	= 1,000
				F(1, 998)	= 53.50
Model	13269.3853	1	13269.3853	Prob > F	= 0.0000
Residual	247525.861	998	248.021905	R-squared	= 0.0509
				Adj R-squared	= 0.0499
Total	260795.247	999	261.056303	Root MSE	= 15.749

ownscore	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
fatherscore	.2370829	.032413	7.31	0.000	.1734773	.3006884
_cons	37.90861	1.672157	22.67	0.000	34.62726	41.18996

Regression: Direct and indirect 2

```
. reg education fatherscore
```

Source	SS	df	MS		
				Number of obs	= 1,000
				F(1, 998)	= 111.01
Model	311.104929	1	311.104929	Prob > F	= 0.0000
Residual	2797.00607	998	2.80261129	R-squared	= 0.1001
				Adj R-squared	= 0.0992
Total	3108.111	999	3.11122222	Root MSE	= 1.6741

education	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
fatherscore	.0363018	.0034455	10.54	0.000	.0295405	.0430631
_cons	1.295213	.1777516	7.29	0.000	.9464035	1.644023

Regression: Direct and indirect 3

```
. reg ownscore education
```

Source	SS	df	MS		
				Number of obs	= 1,000
				F(1, 998)	= 447.54
Model	80742.8091	1	80742.8091	Prob > F	= 0.0000
Residual	180052.437	998	180.413264	R-squared	= 0.3096
				Adj R-squared	= 0.3089
Total	260795.247	999	261.056303	Root MSE	= 13.432

ownscore	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
education	5.096871	.2409273	21.16	0.000	4.624089	5.569653
_cons	33.87079	.8556481	39.58	0.000	32.19171	35.54986

Regression: Direct and indirect 4

```
. reg ownscore education fatherscore
```

Source	SS	df	MS		
Model	81453.7212	2	40726.8606	Number of obs	= 1,000
				F(2, 997)	= 226.41
Residual	179341.525	997	179.881169	Prob > F	= 0.0000
				R-squared	= 0.3123
				Adj R-squared	= 0.3109
Total	260795.247	999	261.056303	Root MSE	= 13.412

ownscore	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
education	4.937369	.2535982	19.47	0.000	4.439722	5.435017
fatherscore	.0578475	.0290984	1.99	0.047	.0007463	.1149486
_cons	31.51367	1.461439	21.56	0.000	28.64582	34.38152

Formula for multiple regression

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 ... + \beta_k X_k + e$$

$$e \sim N(0, \sigma)$$

- Interpretation of β_j
  - How much $\hat{Y}$ changes for a 1-unit in X_j holding all other values constant
  - The estimated effect on Y of a 1-unit change in X_j, "controlling for" or "taking account" of all the other Xs

Predictions

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 ... + \beta_k X_k$$

- Enter values for all X variables to get a prediction for those values
- If we increase X_i by 1, holding all others the same, $\hat{Y}$ changes by β_i

Simplest example

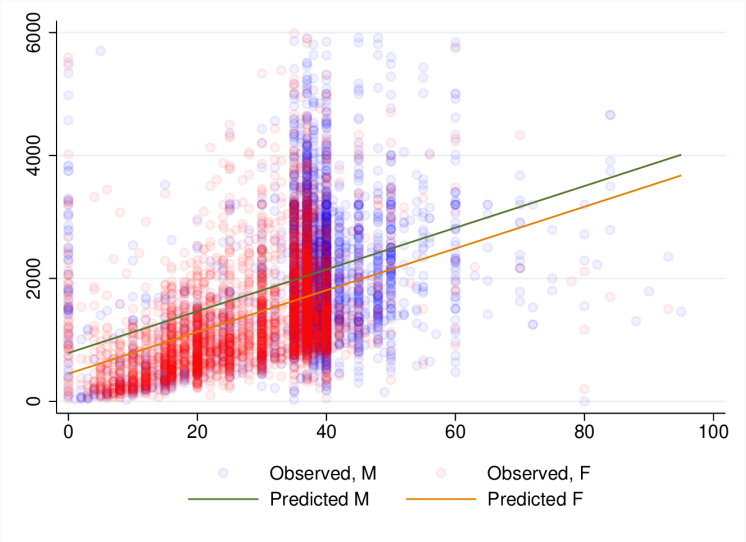
- Simplest multiple regression model adds a binary variable to a model with a continuous X

```
. reg income hours i.sex
```

Source	SS	df	MS		
				Number of obs	= 7,945
				F(2, 7942)	= 794.96
Model	1.8935e+09	2	946761687	Prob > F	= 0.0000
Residual	9.4586e+09	7,942	1190962.07	R-squared	= 0.1668
				Adj R-squared	= 0.1666
Total	1.1352e+10	7,944	1429021.17	Root MSE	= 1091.3

income	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
hours	33.96065	1.123629	30.22	0.000	31.75804	36.16326
sex						
female	-337.0889	26.44232	-12.75	0.000	-388.9228	-285.255
_cons	787.1759	45.73595	17.21	0.000	697.5214	876.8304

Predicted lines: one for each value of sex



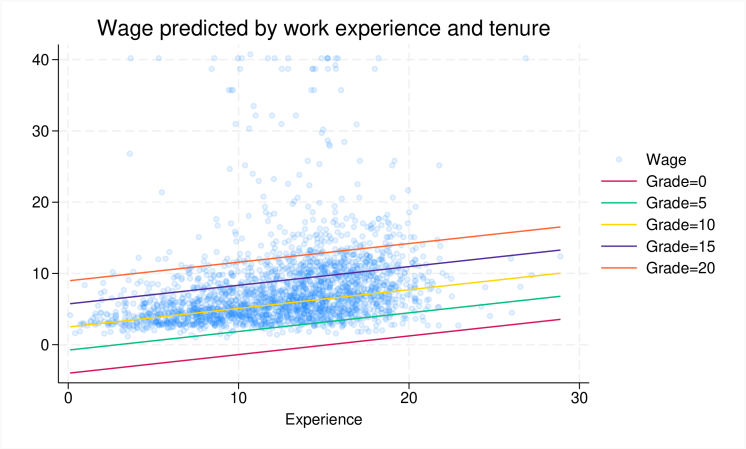
More general 2 X-variable example

```
. reg wage  ttl_exp grade
```

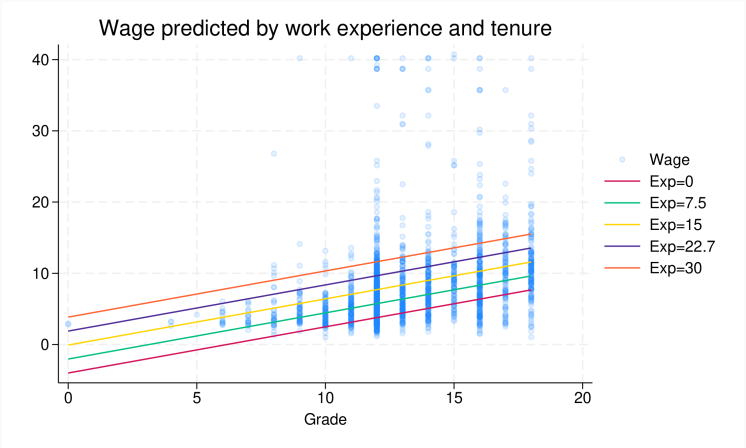
Source	SS	df	MS		
				Number of obs	= 2,244
				F(2, 2241)	= 194.77
Model	11010.6	2	5505.3	Prob > F	= 0.0000
Residual	63343.7305	2,241	28.2658325	R-squared	= 0.1481
				Adj R-squared	= 0.1473
Total	74354.3305	2,243	33.1495009	Root MSE	= 5.3166

wage	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
ttl_exp	.2616056	.0248373	10.53	0.000	.2128992	.310312
grade	.6483343	.045426	14.27	0.000	.5592528	.7374158
_cons	-4.002059	.6245962	-6.41	0.000	-5.226906	-2.777211

Effect of experience on wage, controlling for grade



Effect of grade on wage, controlling for experience



See https://teaching.sociology.ul.ie/so5032/ttlgrade.html

Residuals

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 ... + \beta_k X_k$$

$$Y = \hat{Y} + e$$

$$e \sim N(0, \sigma)$$

- Mean of zero
- Standard deviation of σ (RMSE)
- Normally distributed
- Should have no structured relationship to X variables

Lecture 4: Summary of multiple regression

R^2

R^2

- R^2: coefficient of multiple determination
- TSS = sum of squared deviation from the mean = $\sum(Y_i - \bar{Y})^2$
- RSS = sum of squared deviation from the regression prediction = $\sum(Y_i - \hat{Y})^2$
- $R^2 = \frac{TSS - RSS}{TSS}$
- Range: 0 (no relationship) to 1 (perfect linear relationship)
- PRE: Proportional Reduction in Error

R^2 and correlation

- In bivariate regression, R^2 is the square of the correlation coefficient between Y and X
- In multiple regression, it is the square of the correlation between Y and $\hat{Y}$
- (In bivariate regression the correlation between X and $\hat{Y}$ is 1)

Lecture 4: Summary of multiple regression

Hypothesis testing

Hypothesis testing: one parameter at a time

- t-test: *abs*$(\hat{\beta}_j / \mathrm{se}_j) > t$
- Interpretation:
  - Null: population value of β is 0; this variable has no influence once the other variables are taken account of

Example

```
. reg income age i.sex
```

Source	SS	df	MS		
				Number of obs	= 959
				F(2, 956)	= 45.72
Model	33922983.9	2	16961492	Prob > F	= 0.0000
Residual	354670636	956	370994.389	R-squared	= 0.0873
				Adj R-squared	= 0.0854
Total	388593620	958	405630.083	Root MSE	= 609.09

income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age	-3.144945	1.083398	-2.90	0.004	-5.271057	-1.018833
sex						
female	-352.678	39.51326	-8.93	0.000	-430.2208	-275.1353
_cons	1035.878	54.58935	18.98	0.000	928.7494	1143.007

Hypothesis testing: all parameters together

- F-test:
  - $\beta_1 = \beta_2 \ldots = \beta_k = 0$
- Null hypothesis: no X variable has an effect once the others are taken care of.
- A "global" test: the null is that there is no relevant variable in the model
- Calculation based on TSS and RSS, but also number of cases and number of parameters estimated
- Uses F distribution (two df parameters: k and n-k-1, k is number of parameters, n the number of cases)

Hypothesis testing: additional parameters

- Delta F-test compares "nested" models
  - Model 1: $\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 ... + \beta_g X_g$
  - Model 1: $\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 ... + \beta_g X_g + \beta_h X_h ... + \beta_k X_k$
- Null hypothesis: $\beta_h = \ldots = \beta_k = 0$
- That is, given the variables already in the model, the additional variables contribute no explanatory power.
- Useful when adding multi-category variables, or related groups of variables

Dummy variables

In regression models we often use "indicator coding" or "dummy coding"

With a two-category variable, we set one category to 0 and the other to 1 and interpret it as the effect of being in the second category (e.g., female) compared with the first.

```
. reg income age i.sex
```

Source	SS	df	MS	Number of obs	=	959
				F(2, 956)	=	45.72
Model	33922983.9	2	16961492	Prob > F	=	0.0000
Residual	354670636	956	370994.389	R-squared	=	0.0873
				Adj R-squared	=	0.0854
Total	388593620	958	405630.083	Root MSE	=	609.09

income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age	-3.144945	1.083398	-2.90	0.004	-5.271057	-1.018833
sex						
female	-352.678	39.51326	-8.93	0.000	-430.2208	-275.1353
_cons	1035.878	54.58935	18.98	0.000	928.7494	1143.007

More than two categories

With more that two categories we create a set of binary variables, "indicator variables" or "dummy variables":

	d1	d2	d3	d4
a	1	0	0	0
b	0	1	0	0
c	0	0	1	0
d	0	0	0	1

For m categories, m-1 dummy variables are sufficient.

We interpret the parameter as the estimated effect of being in that category relative to the omitted or "reference" category.

Stata handles this automatically with the `i.` prefix.

Example

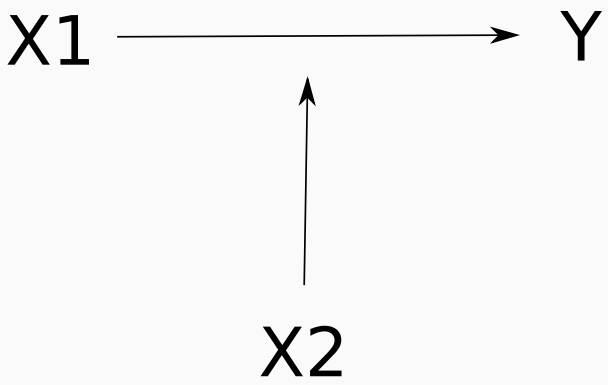
```
. reg income age i.sex i.qual
```

Source	SS	df	MS		
Model	85960604.5	5	17192120.9	Number of obs	= 959
				F(5, 953)	= 54.14
Residual	302633015	953	317558.253	Prob > F	= 0.0000
				R-squared	= 0.2212
				Adj R-squared	= 0.2171
Total	388593620	958	405630.083	Root MSE	= 563.52

income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
age	-.3897295	1.04777	-0.37	0.710	-2.445933	1.666474
sex						
female	-336.9623	36.75947	-9.17	0.000	-409.1011	-264.8234
qual						
A-level, other sub-d..	-459.9208	78.54165	-5.86	0.000	-614.0554	-305.7862
O-level, commercial,..	-701.695	77.16016	-9.09	0.000	-853.1185	-550.2716
Sub-O-level, no qual	-864.9695	76.41768	-11.32	0.000	-1014.936	-715.0032
_cons	1563.508	81.83797	19.10	0.000	1402.904	1724.111

Interactions

- An interaction effect is where the effect of one variable on Y changes depending on the value of another



Income, hours and gender

```
. reg income hours i.sex
```

Source	SS	df	MS		
Model	1.8935e+09	2	946761687	Number of obs	= 7,945
Residual	9.4586e+09	7,942	1190962.07	F(2, 7942)	= 794.96
Total	1.1352e+10	7,944	1429021.17	Prob > F	= 0.0000
				R-squared	= 0.1668
				Adj R-squared	= 0.1666
				Root MSE	= 1091.3

income	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
hours	33.96065	1.123629	30.22	0.000	31.75804	36.16326
sex						
female	-337.0889	26.44232	-12.75	0.000	-388.9228	-285.255
_cons	787.1759	45.73595	17.21	0.000	697.5214	876.8304

For men

```
. reg income hours if sex==1
```

Source	SS	df	MS	Number of obs	=	3,818
				F(1, 3816)	=	204.70
Model	344180174	1	344180174	Prob > F	=	0.0000
Residual	6.4162e+09	3,816	1681398.47	R-squared	=	0.0509
				Adj R-squared	=	0.0507
Total	6.7604e+09	3,817	1771128.3	Root MSE	=	1296.7

income	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
hours	28.71923	2.007313	14.31	0.000	24.78372	32.65474
_cons	983.9722	78.23438	12.58	0.000	830.587	1137.357

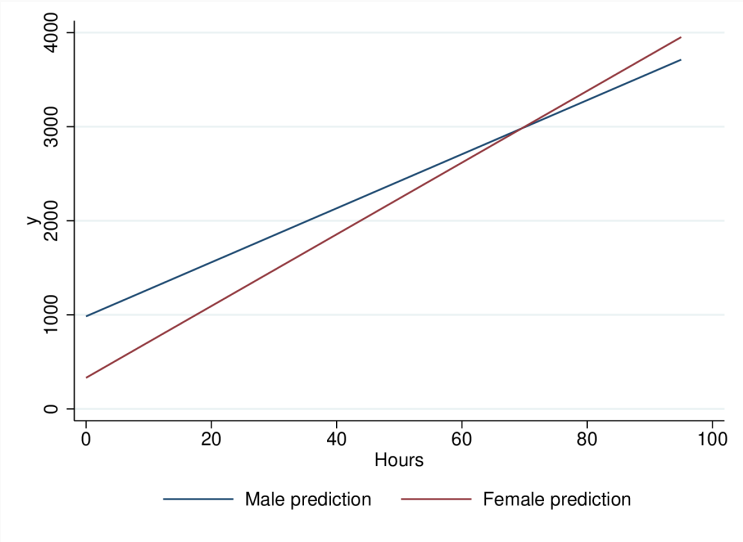
For women

```
. reg income hours if sex==2
```

Source	SS	df	MS	Number of obs	=	4,127
				F(1, 4125)	=	1043.34
Model	764315243	1	764315243	Prob > F	=	0.0000
Residual	3.0218e+09	4,125	732568.614	R-squared	=	0.2019
				Adj R-squared	=	0.2017
Total	3.7862e+09	4,126	917634.7	Root MSE	=	855.9

income	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
hours	38.11874	1.180121	32.30	0.000	35.80507	40.43241
_cons	330.7275	36.40158	9.09	0.000	259.3607	402.0942

Different effects



Interaction in regression

- We can capture interaction effects with a regression model of this form:

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2$$

- That is, a 1-unit increase in X_1 leads to a $\beta_1 + \beta_3 X2$ increase in $\hat{Y}$
- Equivalently, a 1-unit increase in X_2 leads to a $\beta_1 + \beta_3 X1$ increase in $\hat{Y}$

Interaction between hours and sex

- Simplest example: one variable is binary

$$\hat{Y}_m = \beta_0 + \beta_1 X_1 + \beta_2 \times 0 + \beta_3 X_1 \times 0$$

$$\hat{Y}_f = \beta_0 + \beta_1 X_1 + \beta_2 \times 1 + \beta_3 X_1 \times 1$$

One-unit increase

If X_1 increases by 1 unit, $\hat{Y}$ changes:

$$\Delta \hat{Y}_m = \beta_1$$

$$\Delta \hat{Y}_f = \beta_1 + \beta_3$$

Stata: by hand

- First create an interaction variable:

```
gen female = sex == 2
gen intvar = hours*female
```

- Then fit the regression:

```
reg income hours female intvar
```

Results

```
. gen female = sex==2

. gen intvar = female*hours

. reg income hours female intvar
```

Source	SS	df	MS	Number of obs	=	7,945
				F(3, 7941)	=	536.82
Model	1.9141e+09	3	638027348	Prob > F	=	0.0000
Residual	9.4381e+09	7,941	1188523.12	R-squared	=	0.1686
				Adj R-squared	=	0.1683
Total	1.1352e+10	7,944	1429021.17	Root MSE	=	1090.2

income	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
hours	28.71923	1.687655	17.02	0.000	25.41098	32.02747
female	-653.2448	80.47524	-8.12	0.000	-810.9974	-495.4921
intvar	9.399515	2.260017	4.16	0.000	4.969287	13.82974
_cons	983.9722	65.7758	14.96	0.000	855.0344	1112.91

Stata's formula syntax

- But more convenient to use Stata's formula syntax

  ```
  reg income c.hours##i.sex
  ```

- `i.sex` means treat `sex` as categorical
- `c.hours#i.sex` creates the interaction between hours (continuous, `c.`) and sex
- `c.hours##i.sex` puts both the interaction and the first order terms in the model

Same results using Stata's formula syntax

```
. reg income c.hours##i.sex
```

Source	SS	df	MS	Number of obs	=	7,945
				F(3, 7941)	=	536.82
Model	1.9141e+09	3	638027348	Prob > F	=	0.0000
Residual	9.4381e+09	7,941	1188523.12	R-squared	=	0.1686
				Adj R-squared	=	0.1683
Total	1.1352e+10	7,944	1429021.17	Root MSE	=	1090.2

income	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
hours	28.71923	1.687655	17.02	0.000	25.41098	32.02747
sex						
female	-653.2448	80.47524	-8.12	0.000	-810.9974	-495.4921
sex#c.hours						
female	9.399515	2.260017	4.16	0.000	4.969287	13.82974
_cons	983.9722	65.7758	14.96	0.000	855.0344	1112.91

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Predictions

Sex	Hrs	β_0	β_1	β_2	β_3	$\hat{y}$
M	0	983.9722	+ 0*28.71923	+ 0*-653.2448	+ 0*0*9.399515	= 983.9722
M	80	983.9722	+ 80*28.71923	+ 0*-653.2448	+ 80*0*9.399515	= 3281.5106
F	0	983.9722	+ 0*28.71923	+ 1*-653.2448	+ 0*1*9.399515	= 330.7274
F	80	983.9722	+ 80*28.71923	+ 1*-653.2448	+ 80*1*9.399515	= 3380.227

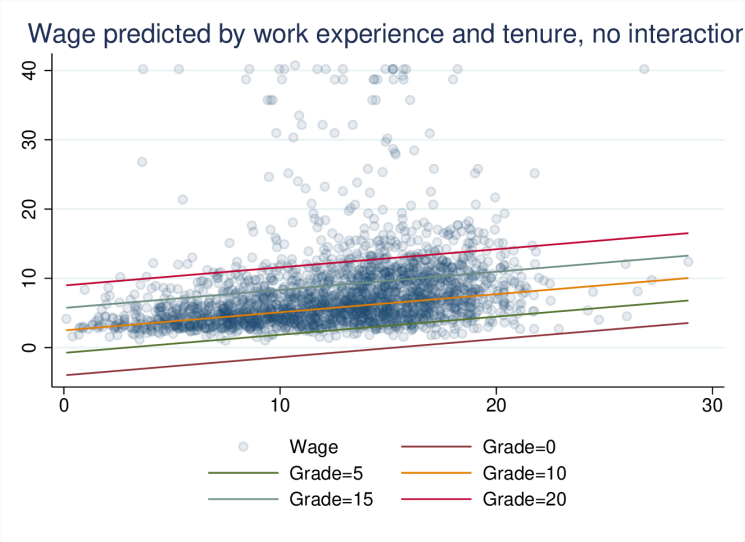
Interactions between two continuous variable

```
. reg wage c.ttl_exp##c.grade
```

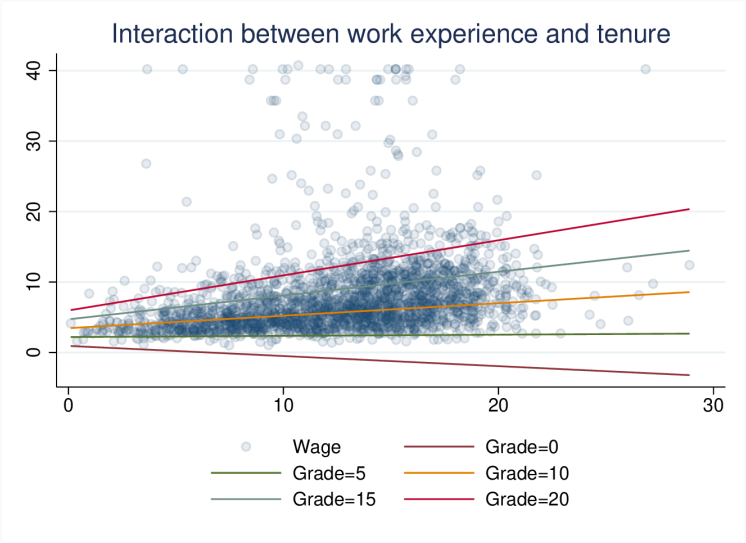
Source	SS	df	MS		
				Number of obs	= 2,244
				F(3, 2240)	= 133.83
Model	11301.2662	3	3767.08872	Prob > F	= 0.0000
Residual	63053.0643	2,240	28.1486894	R-squared	= 0.1520
				Adj R-squared	= 0.1509
Total	74354.3305	2,243	33.1495009	Root MSE	= 5.3055

wage	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
ttl_exp	-.143543	.1284932	-1.12	0.264	-.3955211	.1084352
grade	.2515455	.1315367	1.91	0.056	-.0064011	.5094921
c.ttl_exp#c.grade	.032074	.0099813	3.21	0.001	.0125005	.0516475
_cons	.933757	1.657647	0.56	0.573	-2.316929	4.184443

Without interaction, predictions for different levels of grade



With interaction



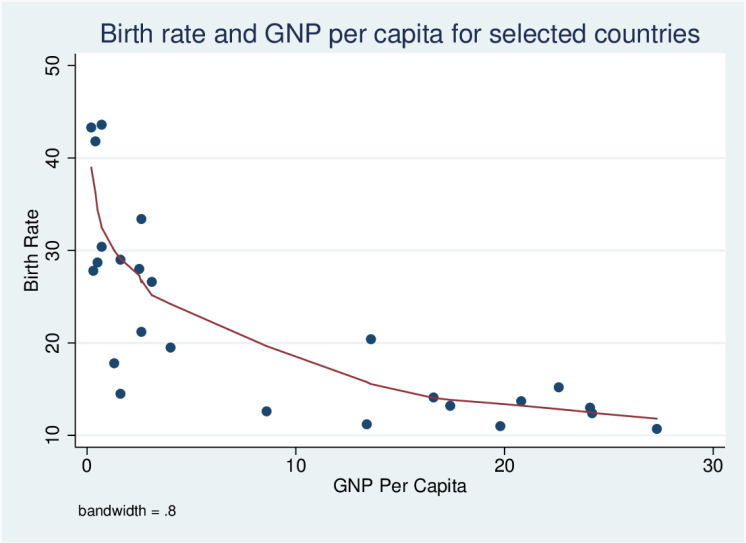
Lecture 5: Interaction and Non-linearity

Non-linear linear regression

Birth rate and GNP example

```
do http://teaching.sociology.ul.ie/so5032/birth
sort gnp
label var bir "Birth Rate"
label var gnp "GNP Per Capita"
lowess bir gnp, title("Birth rate and GNP per capita for selected countries")
```

Nonlinear plot



Get linear relationship

```
reg bir gnp
```

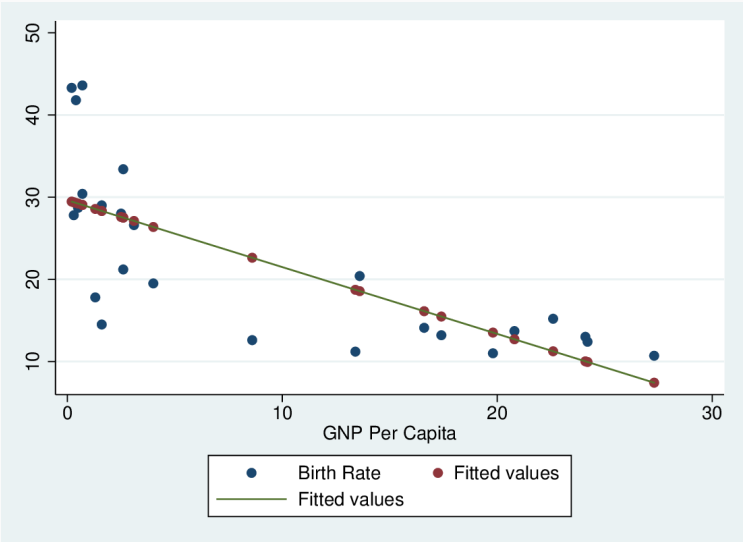
```
. reg bir gnp
```

Source	SS	df	MS	Number of obs	=	25
				F(1, 23)	=	27.52
Model	1450.2603	1	1450.2603	Prob > F	=	0.0000
Residual	1212.02523	23	52.696749	R-squared	=	0.5447
				Adj R-squared	=	0.5249
Total	2662.28552	24	110.928563	Root MSE	=	7.2593

bir	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
gnp	-.8133082	.155033	-5.25	0.000	-1.134018	-.4925981
_cons	29.6227	2.037416	14.54	0.000	25.40798	33.83742

```
predict plin
scatter bir plin gnp|| line plin gnp
```

Linear plot


50
40
30
20
10
0
10
20
30
GNP Per Capita
Birth Rate
Fitted values
Fitted values

Quadratic

Linear regression doesn't fit well

Clearly, as GNP rises BIR falls, but the rate of fall declines

Let's try quadratic:

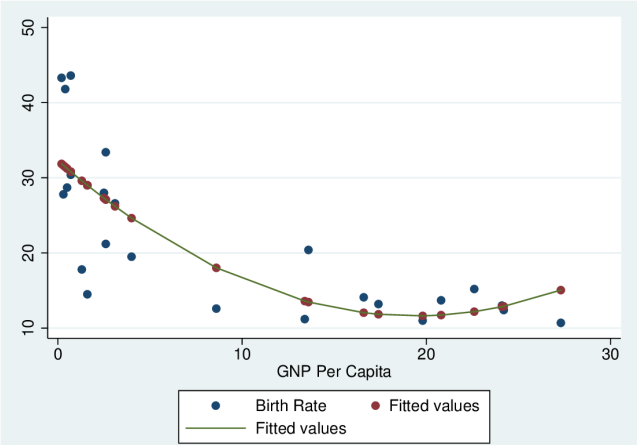
```
. reg bir c.gnp##c.gnp
```

Source	SS	df	MS	Number of obs	=	25
				F(2, 22)	=	18.39
Model	1665.82856	2	832.914278	Prob > F	=	0.0000
Residual	996.456968	22	45.2934985	R-squared	=	0.6257
				Adj R-squared	=	0.5917
Total	2662.28552	24	110.928563	Root MSE	=	6.73

bir	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
gnp	-2.130192	.6205087	-3.43	0.002	-3.417048	-.8433351
c.gnp#c.gnp	.0549243	.0251762	2.18	0.040	.0027121	.1071366
_cons	32.27852	2.247195	14.36	0.000	27.61812	36.93892

Quatratic plot

```
predict pquad
scatter bir pquad gnp|| line pquad gnp
```



Let's try square root of GNP:

```
. gen sqg = sqrt(gnp)

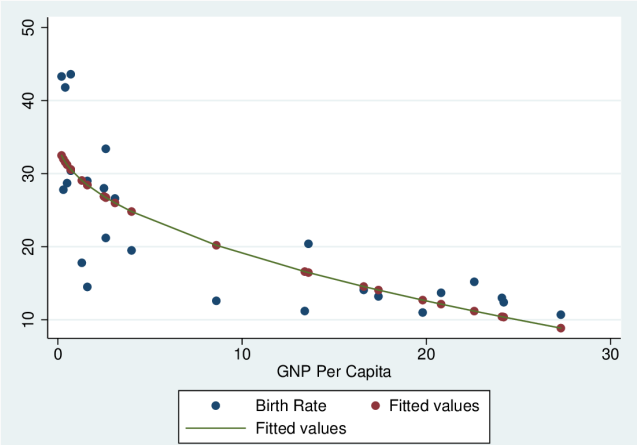
. reg bir sqg
```

Source	SS	df	MS	Number of obs	=	25
				F(1, 23)	=	39.44
Model	1681.66084	1	1681.66084	Prob > F	=	0.0000
Residual	980.624685	23	42.6358559	R-squared	=	0.6317
				Adj R-squared	=	0.6156
Total	2662.28552	24	110.928563	Root MSE	=	6.5296

bir	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
sqg	-4.945487	.7874579	-6.28	0.000	-6.574468	-3.316506
_cons	34.70314	2.391073	14.51	0.000	29.75683	39.64946

√GNP plot

```
predict psqrt
scatter bir psqrt gnp|| line psqrt gnp
```



log(GNP)

Let's try the log of GNP:

```
. gen lgg = log(gnp)

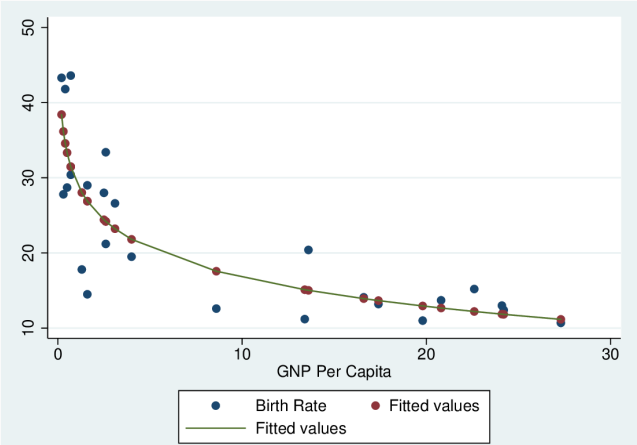
. reg bir lgg
```

Source	SS	df	MS	Number of obs	=	25
				F(1, 23)	=	54.84
Model	1875.68482	1	1875.68482	Prob > F	=	0.0000
Residual	786.600705	23	34.2000307	R-squared	=	0.7045
				Adj R-squared	=	0.6917
Total	2662.28552	24	110.928563	Root MSE	=	5.8481

bir	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lgg	-5.542152	.748362	-7.41	0.000	-7.090257	-3.994047
_cons	29.49466	1.53576	19.21	0.000	26.3177	32.67162

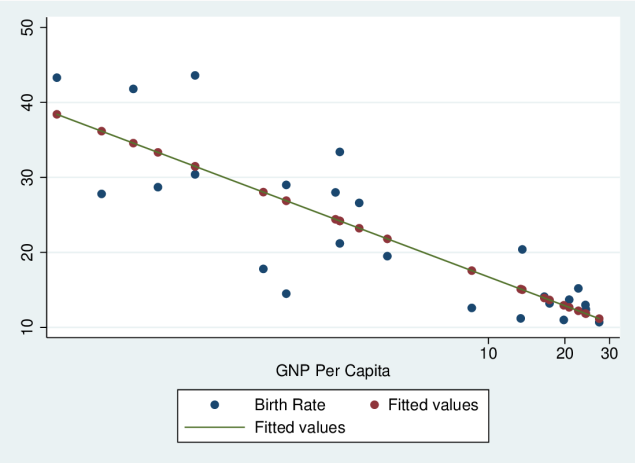
log(GNP) plot

```
predict plog
scatter bir plog gnp|| line plog gnp
```



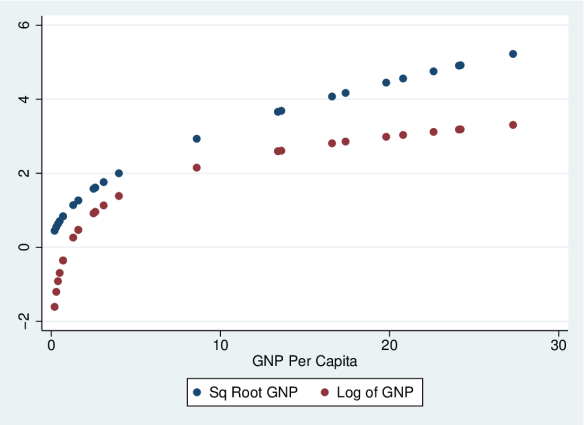
Log-scale plot

```
scatter bir plog gnp, xscale(log)|| line plog gnp, xscale(log)
```



Square root and log compared

```
label var sqg "Sq Root GNP"
label var lg "Log of GNP"
scatter sqg lg gnp
```



Residuals

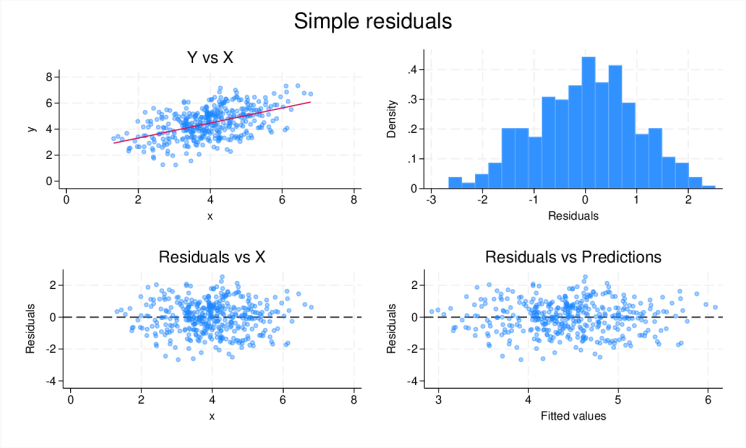
$$Y = b_0 + b_1 X_1 + ... + b_k X_k + e$$

$$e \sim N(0, \sigma)$$

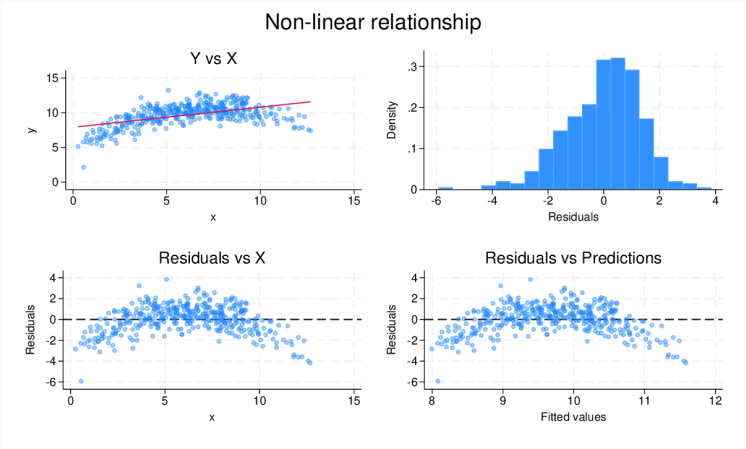
Characteristics

- Residuals will
  - have mean 0
  - be as small as possible
  - have no linear relationship to X variables
- Residuals should
  - be approximately normally distributed (symmetric is often enough)
  - not have a non-linear relationship to any X variable
  - have a constant spread, that is not related to X or Y values
- If correlated with variables not in the model, perhaps those variables should be included

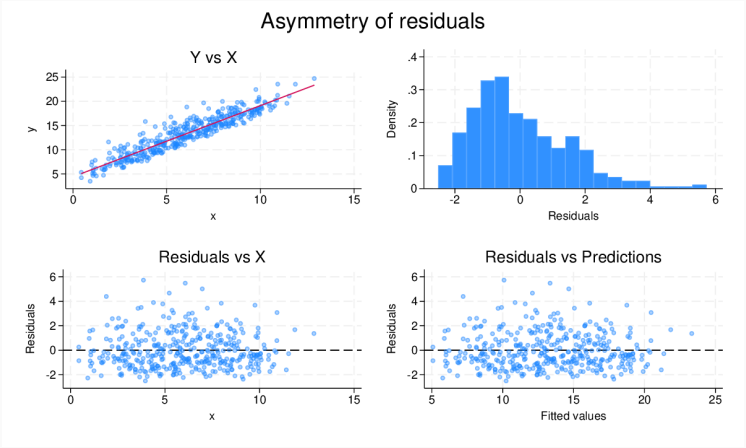
Examining residuals: ideal



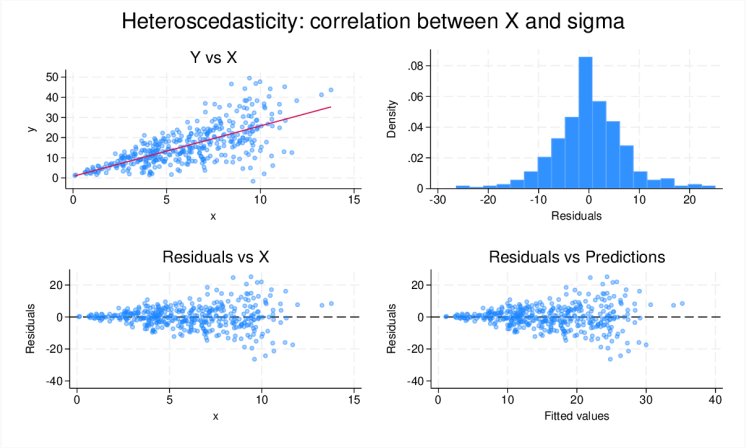
Examining residuals: Non-linear



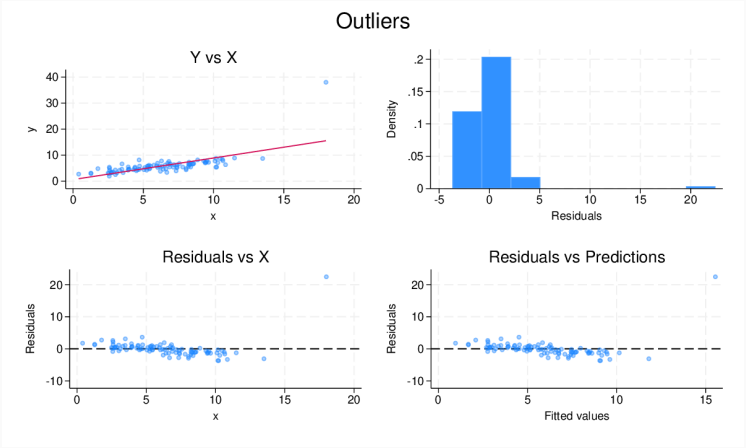
Examining residuals: asymmetric



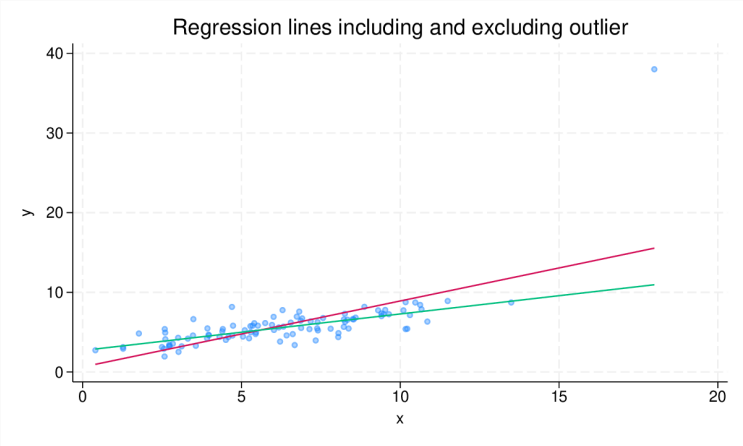
Examining residuals: heteroscedasticity



Examining residuals: Spotting outliers



Examining residuals: Influence of outliers



Lecture 6: Residuals and Influence

Influence

Outliers may have undue influence

- `dfbeta`
- Cook's distance

DFBETA

- For each variable in the regression, for each case
- The effect of dropping that case on that variable
- Scaled by the standard error:

$$\frac{b - b^*}{SE}$$

Cook's Distance

- A single number summarising each case's overall influence
- A scaled sum of changes in predicted Y

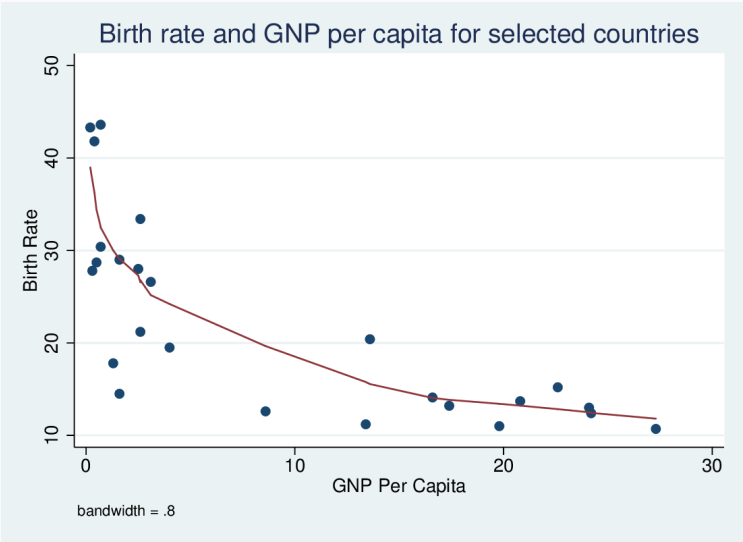
Outlier interactive app

https://teaching.sociology.ul.ie/apps/influence/

Birth rate and GNP example

```
do http://teaching.sociology.ul.ie/so5032/birth
sort gnp
label var bir "Birth Rate"
label var gnp "GNP Per Capita"
lowess bir gnp, title("Birth rate and GNP per capita for selected countries'
```

Nonlinear plot



Get linear relationship

```
reg bir gnp
```

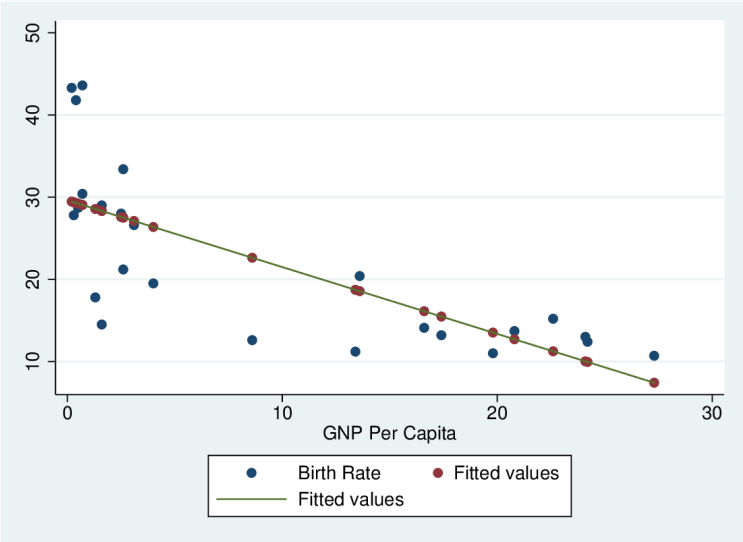
```
. reg bir gnp
```

Source	SS	df	MS		
Model	1450.2603	1	1450.2603	Number of obs	= 25
Residual	1212.02523	23	52.696749	F(1, 23)	= 27.52
				Prob > F	= 0.0000
Total	2662.28552	24	110.928563	R-squared	= 0.5447
				Adj R-squared	= 0.5249
				Root MSE	= 7.2593

bir	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
gnp	-.8133082	.155033	-5.25	0.000	-1.134018	-.4925981
_cons	29.6227	2.037416	14.54	0.000	25.40798	33.83742

```
predict plin
scatter bir plin gnp|| line plin gnp
```

Linear plot


50
40
30
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10
0
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30
GNP Per Capita
Birth Rate
Fitted values
Fitted values

Quadratic

Linear regression doesn't fit well

Clearly, as GNP rises BIR falls, but the rate of fall declines

Let's try quadratic:

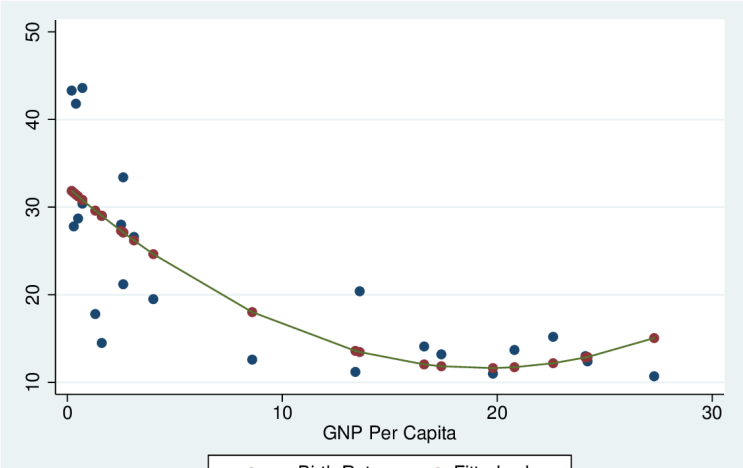
```
reg bir c.gnp##c.gnp
```

```
. reg bir c.gnp##c.gnp
      Source |       SS           df       MS      Number of obs   =        25
-------------+----------------------------------   F(2, 22)        =     18.39
       Model |  1665.82856         2  832.914278   Prob > F        =    0.0000
    Residual |  996.456968        22  45.2934985   R-squared       =    0.6257
-------------+----------------------------------   Adj R-squared   =    0.5917
       Total |  2662.28552        24  110.928563   Root MSE        =      6.73

------------------------------------------------------------------------------
         bir |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gnp |   -2.130192   .6205087    -3.43   0.002    -3.417048   -.8433351
```

Quatratic plot

```
predict pquad
scatter bir pquad gnp|| line pquad gnp
```



Let's try square root of GNP:

```
gen sqg = sqrt(gnp)
reg bir sqg
```

```
. gen sqg = sqrt(gnp)

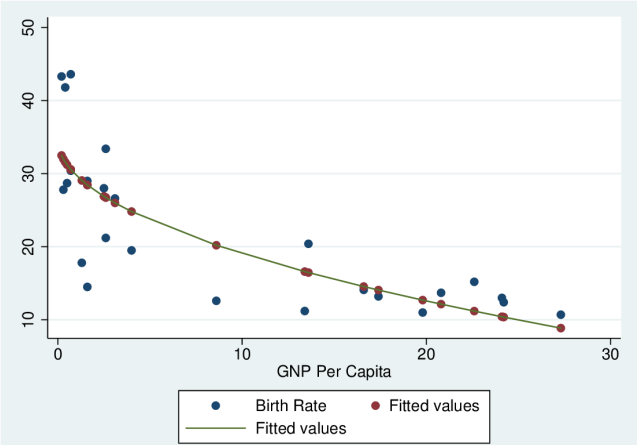
. reg bir sqg
```

Source	SS	df	MS	Number of obs	=	25
				F(1, 23)	=	39.44
Model	1681.66084	1	1681.66084	Prob > F	=	0.0000
Residual	980.624685	23	42.6358559	R-squared	=	0.6317
				Adj R-squared	=	0.6156
Total	2662.28552	24	110.928563	Root MSE	=	6.5296

bir	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
sqg	-4.945487	.7874579	-6.28	0.000	-6.574468	-3.316506
_cons	34.70314	2.391073	14.51	0.000	29.75683	39.64946

$\sqrt{GNP}$ plot

```
predict psqrt
scatter bir psqrt gnp|| line psqrt gnp
```



log(GNP)

Let's try the log of GNP:

```
gen lgg = log(gnp)
reg bir lgg
```

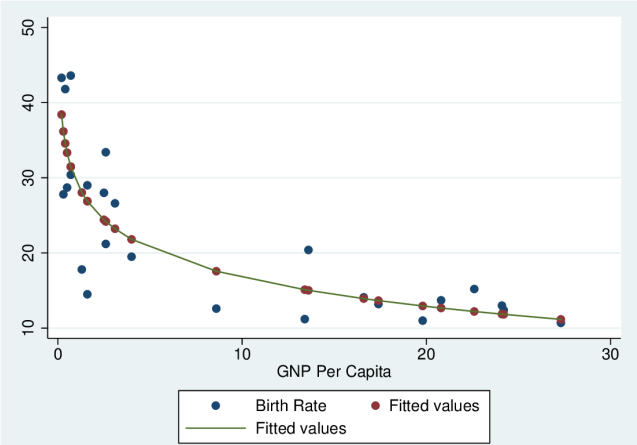
```
. gen lgg = log(gnp)
. reg bir lgg
```

Source	SS	df	MS	Number of obs	=	25
				F(1, 23)	=	54.84
Model	1875.68482	1	1875.68482	Prob > F	=	0.0000
Residual	786.600705	23	34.2000307	R-squared	=	0.7045
				Adj R-squared	=	0.6917
Total	2662.28552	24	110.928563	Root MSE	=	5.8481

bir	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lgg	-5.542152	.748362	-7.41	0.000	-7.090257	-3.994047
_cons	29.49466	1.53576	19.21	0.000	26.3177	32.67162

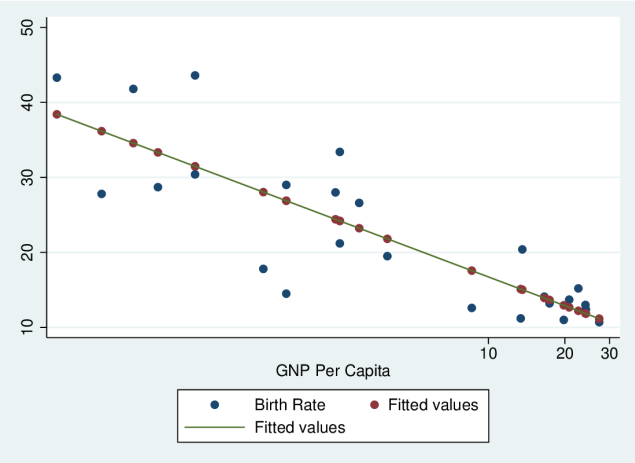
log(GNP) plot

```
predict plog
scatter bir plog gnp|| line plog gnp
```



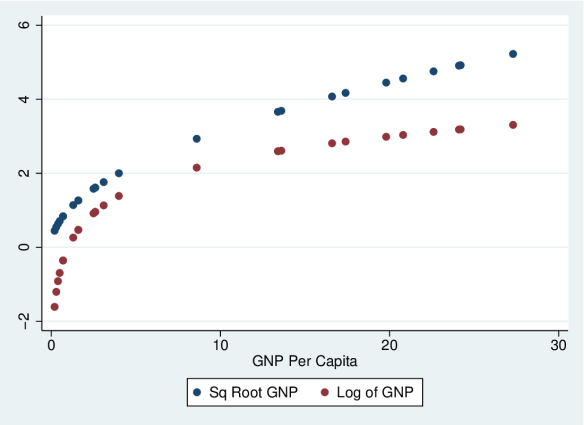
Log-scale plot

```
scatter bir plog gnp, xscale(log)|| line plog gnp, xscale(log)
```



Square root and log compared

```
label var sqg "Sq Root GNP"
label var lg "Log of GNP"
scatter sqg lg gnp
```



Lecture 7: Logs and log regression

Logarithms

Logarithms

Logarithms allow us to move between multiplicative equations and additive ones.

Logs are defined relative to a base number. If we take 10 as the base then $y = log_{10}(x)$ means $10^x = y$.

It's easy to calculate the log of powers of 10:

log(10) = 1	$10^1 = 10$
log(100) = 2	$10^2 = 100$
log(1000) = 3	$10^3 = 1000$
log(1000000) = 6	$10^6 = 1000000$

10^0 is defined as 1, so the log of 1 is zero.

From 0 to 1

For numbers between 1 and 0, logs are negative

$$\frac{1}{10} = 10^{-1} \quad \log(0.1) = -1$$
$$\frac{1}{100} = 10^{-2} \quad \log(0.01) = -2$$
$$\frac{1}{1000} = 10^{-3} \quad \log(0.001) = -3$$

The $\log_{10}$ of powers of 10 are integers, but we can raise 10 to non-integer powers too, to get the log of any number greater than zero. For instance, $10^{2.09}$ is 123, so the log of 123 is 2.09.

Multiply by adding

We can see with round powers of 10 than using logs we can move between multiplication and addition:

$100 \times 1000 = 100000$

$10^2 \times 10^3 = 10^5 = 10^{2+3}$

Calculate A × B

Thus do calculate A × B we do as follows:

- Calculate log(A)
- Calclate log(B)
- Calculate log(C) = log(A) + log(B)
- Take the anti-log of log(C), i.e., $10^{\log(C)} = C$

Example

Multiply 12345 by 67890

log(12345) = 9.421

log(67890) = 11.126

9.421 + 11.126 = 20.547

$10^{20.547}$ = 838102050

An application

If you have a certain quantity (e.g., money in a bank account), whose value increases by a constant proportion every year, its value in any year depends on a multiplicative relationship.

Let's say the increases is α (i.e., a 10% increase means α = 1.1)

Compound interest

Year 0	100
Year 1	$100 \times \alpha$
Year 2	$100 \times \alpha \times \alpha$
Year 3	$100 \times \alpha \times \alpha \times \alpha$
Year 4	$100 \times \alpha \times \alpha \times \alpha \times \alpha$
Year 5	$100 \times \alpha \times \alpha \times \alpha \times \alpha \times \alpha$

In short, the value in year t is $100 \times \alpha^t$

$$y_t = 100 \times \alpha^t$$

Constant proportional increase

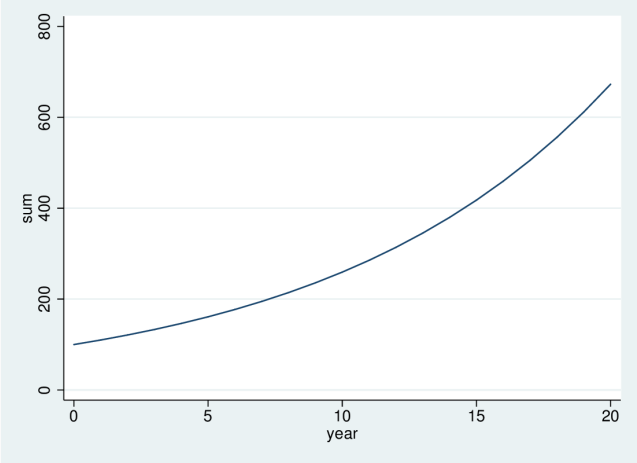


Figure 1: A constant proportional increase

Convert to logs

But if we convert to logs we can calculate it as follows

$$log(y_t) = log(100) + t \times log(\alpha)$$

In other words, rather than multiplying by α every year, we add log(α).

Plot

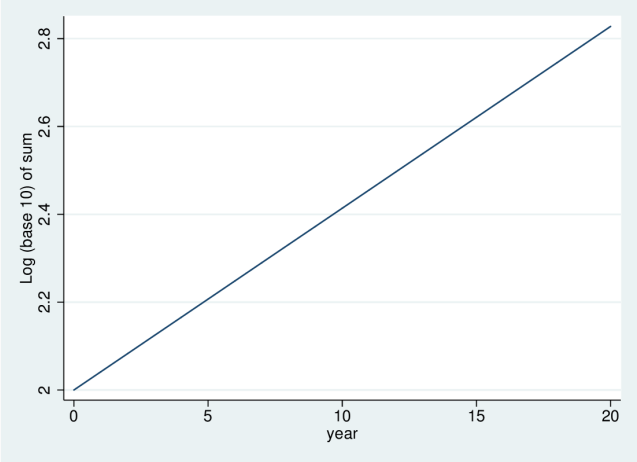


Figure 2: Taking the base-10 log of the sum: a straight line

Straight line

This gives a straight line relationship (see Fig 2).

Thus we can use logs to move between multiplicative and additive (straight-line) relationships.

Other bases

Logs to the base 10 are easy to understand, but the base number need not be 10. A log to the base n is defined thus:

$$y = log_n(x) \Leftrightarrow n^y = x$$

Natural logs

Computer scientists often use $\log_2$, but the most common log base is the special number $e \approx 2.7183$. This has some special mathematical properties that make certain calculations easier.

Logs to base e are called natural logs, often written ln(x) etc:

$$y = ln(x) \Leftrightarrow e^y = x$$

See Fig 3, which shows that the natural log also gives a straight line.

Natural log straight line

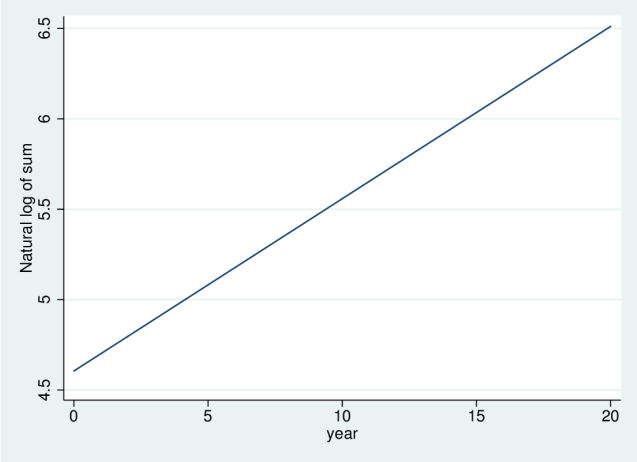


Figure 3: Taking the natural log of the sum: also a straight line

Natural log

- Fig 4 shows the natural log of X from 0.1 (-2.303) to 100 (4.605).
- For X = 1, the log is 0.
- As X approaches 0, the log falls faster and faster.
- As X rises above 1, the log rises, but more slowly as it goes.
- Note that the log rises from X = 5 to 10 as much as it does from X = 40 to 80.

X vs ln(X)

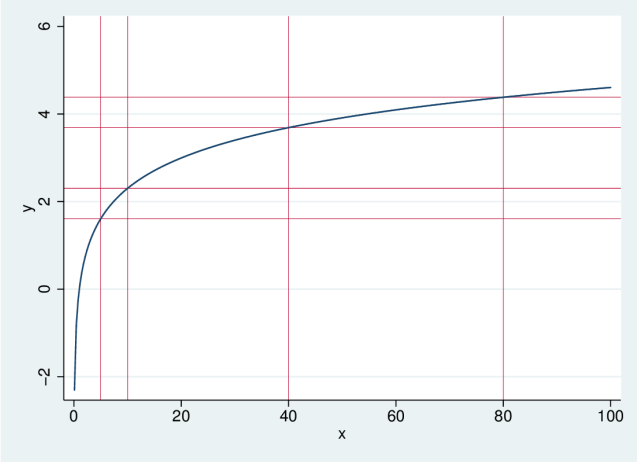


Figure 4: The natural log of X for X from 0.1 to 100

Lecture 7: Logs and log regression

Early pandemic: exponential curves

Logs and COVID-19

- In the early stage of an epidemic, infections tend to increase at a steady rate
- On average each infected person infects others at a given rate, e.g., one person every four days
- So numbers of cases tend to rise at a steady percentage
  - New infections are proportional to existing infections
  - 100 today means 125 tomorrow, 156 the next day, etc.

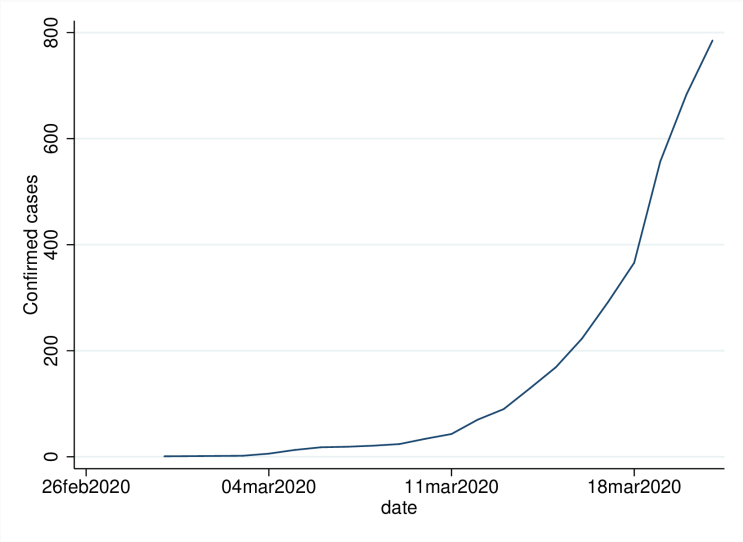
Confirmed cases in Ireland

If we look at the raw number of cases in Ireland:

- it starts off very low
- stays there for a while
- but then starts rising
- and rising faster and faster

```
line cases date
```

Confirmed cases in Ireland



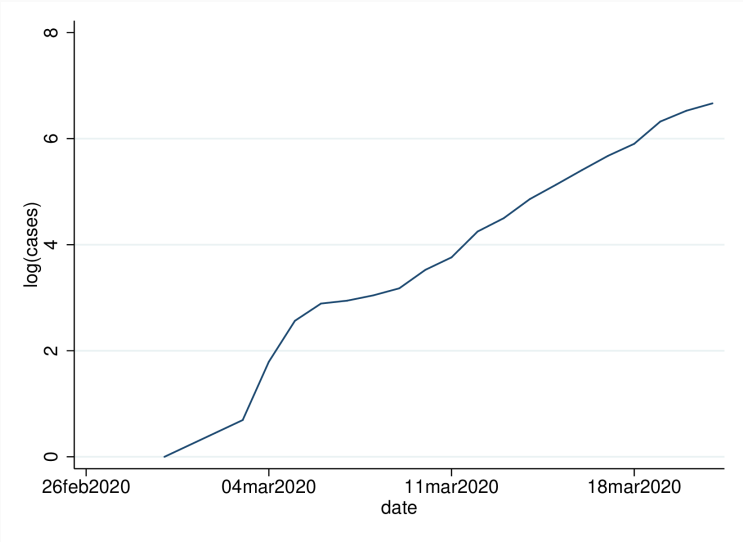
Log cases

If we plot the log of the cases we see a different picture

- wobbly to begin with
- then approximating a straight line

```
gen lcases = log(cases)
line lcases date
```

Log cases


log(cases)
0
2
4
6
8
26feb2020
04mar2020
11mar2020
18mar2020
date

Log cases: straight => exponential

A straight line in logs means `log(ncases)` increases by more or less a set amount very day

That means `ncases` rises by a set proportion every day: exponential rise

Exponential: even if it starts small, if given long enough, will get very very big!

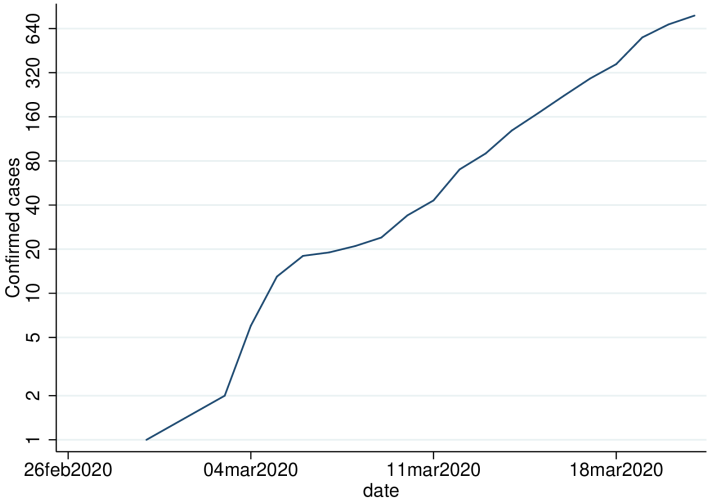
Log scale, real cases

We can graph `log(cases)` but we can also graph `cases` with a Y log-scale

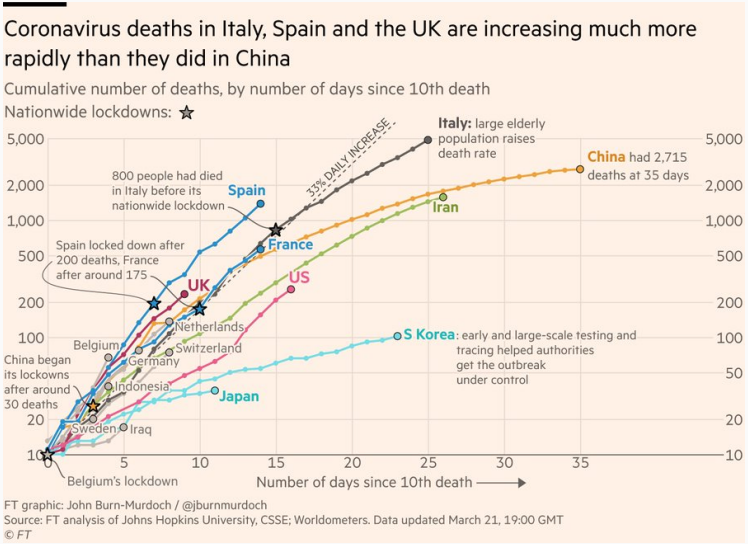
```
line cases date, yscale(log) ylabel(1 2 5 10 20 40 80 160 320 640)
```

This gives the advantages of the logging while retaining the real numbers on the axis

Log scale, real cases



Log-scale graphic in the wild



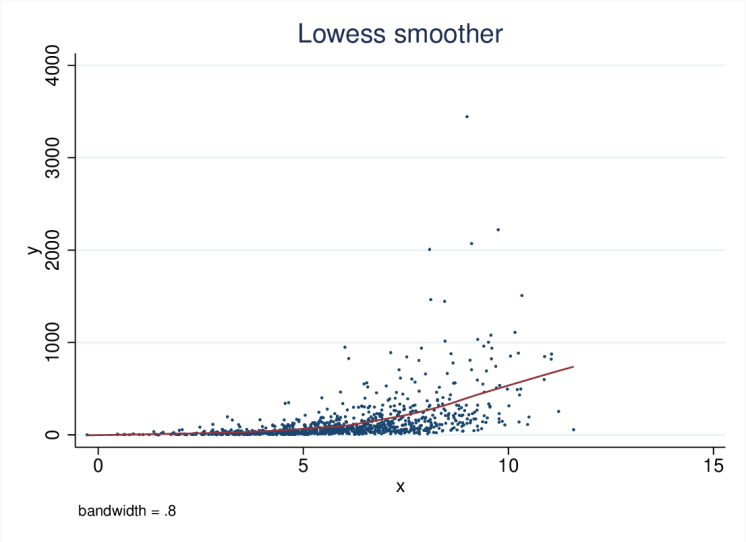
Lecture 7: Logs and log regression

Log regression

Multiplicative relationship

- Where the underlying relationship is multiplicative, linear regression doesn't work well
- Implies an additive increase where a multiplicative one is better
- If we take the log of the dependent variable:
  - better estimates
  - often cures heteroscedasticity

Simulation: Y increases 65% for X +1



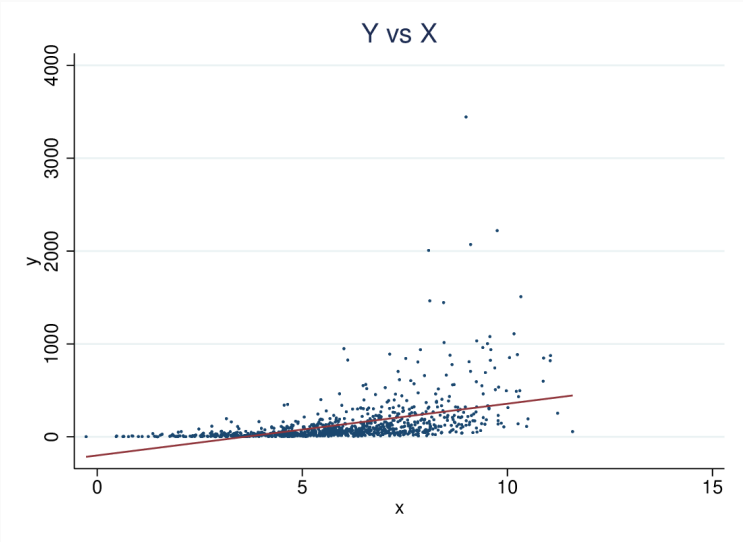
Linear regression

```
. reg y x
```

Source	SS	df	MS		
				Number of obs	= 1,000
				F(1, 998)	= 274.71
Model	12181477.5	1	12181477.5	Prob > F	= 0.0000
Residual	44253675.2	998	44342.3599	R-squared	= 0.2158
				Adj R-squared	= 0.2151
Total	56435152.7	999	56491.6443	Root MSE	= 210.58

y	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
x	55.69088	3.360033	16.57	0.000	49.09734	62.28442
_cons	-200.7041	20.95566	-9.58	0.000	-241.8263	-159.5819

Predictions



Log(Y)

```
. gen ly = log(y)

. reg ly x
```

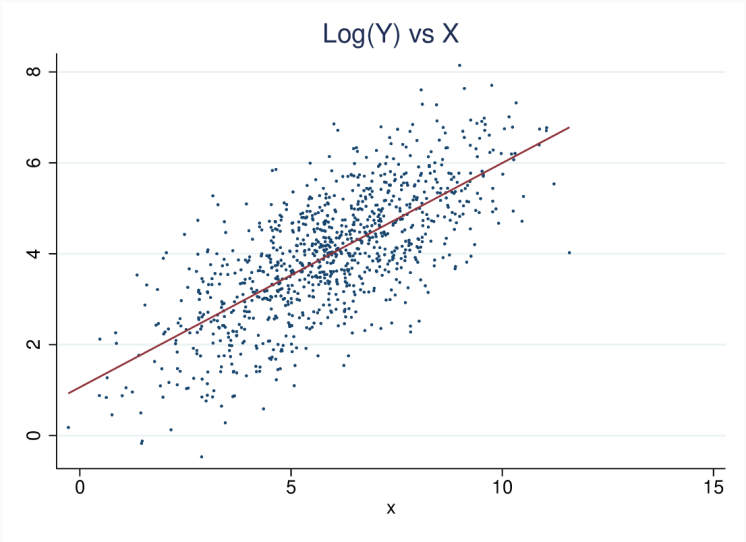
Source	SS	df	MS	Number of obs	=	1,000
				F(1, 998)	=	1032.66
Model	956.12538	1	956.12538	Prob > F	=	0.0000
Residual	924.030142	998	.925881905	R-squared	=	0.5085
				Adj R-squared	=	0.5080
Total	1880.15552	999	1.88203756	Root MSE	=	.96223

ly	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
x	.4933914	.0153537	32.14	0.000	.4632622	.5235205
_cons	1.062305	.0957568	11.09	0.000	.8743972	1.250213

Interpretation

- For a 1 unit change in X, $log(\hat{Y})$ rises by 0.4933914
- Thus for a 1 unit change in X, Y rises by $e^{0.4933914} = 1.638$
- $e^{0.4933914}$ is the antilog of 0.4933914

Predictions



Predicted values

- Where the dependent variable is logged the prediction of the Y value is not simply the anti-log of the predicted log(Y)
- When we take the anti-log we must take account of the fact that residuals above the line expand by more than residuals below the line
- Thus a small correction

$$\hat{log(Y)} = a + bX$$

$$\hat{Y} = e^{\hat{log(Y)}} * e^{\mathrm{RMSE}^2/2}$$

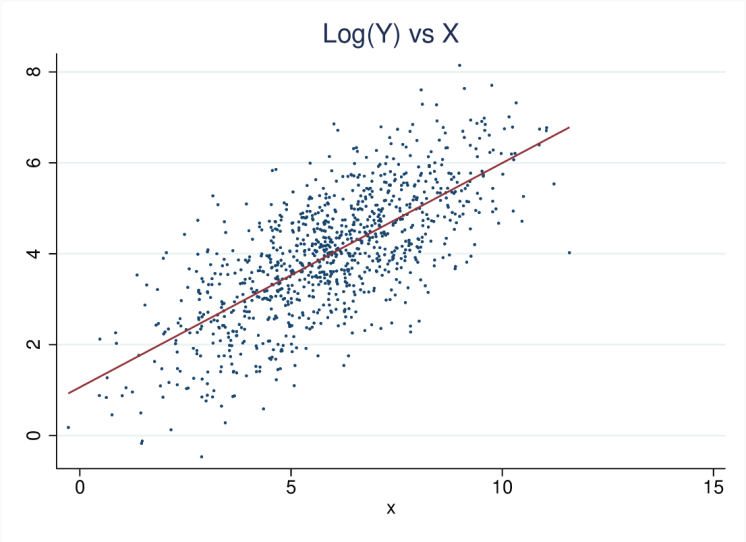
- where RMSE is the standard deviation of the regression

Calculations

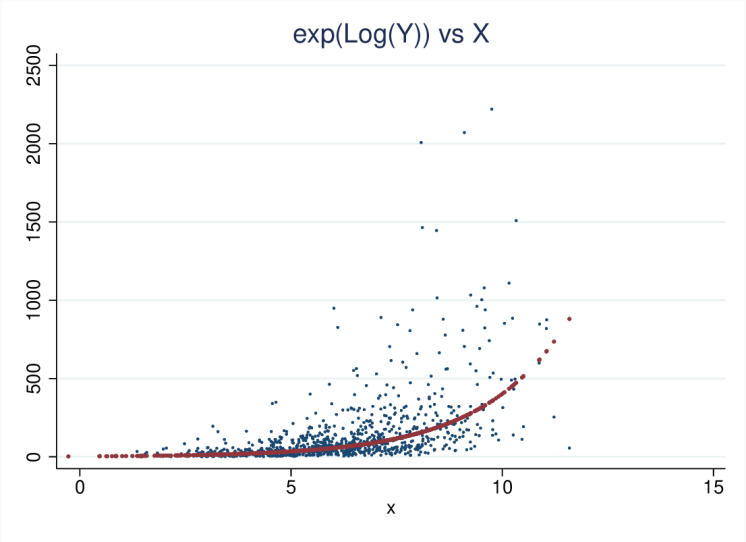
```
gen ly = log(y)
reg ly x

predict lyhat
gen elyh = exp(lyhat)
gen elyh2 = elyh * exp(rmse^2/2)
```

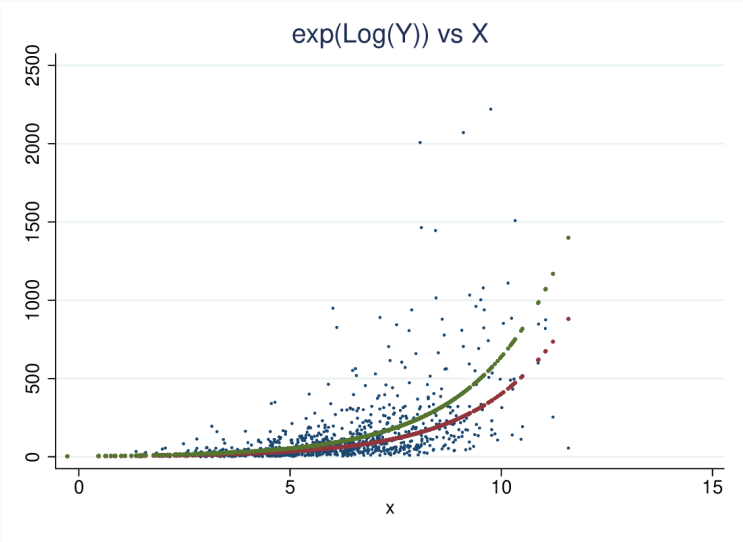
Predictions: predict log(Y) on log scale



Predictions: only $e^{log(\hat{Y})}$



Predictions: with correction



Predicting COVID-19

- We can apply log regression to the COVID-19 data
- A straight line on a log scale means a constant proportional increase.
- We can estimate this increase, regressing log(cases) on date.
- The slope, b, is the amount by which $\log \hat{\text{cases}}$ rises per day
- e^b is then the multiplier by which `cases` rises per day

```
reg lcases date
```

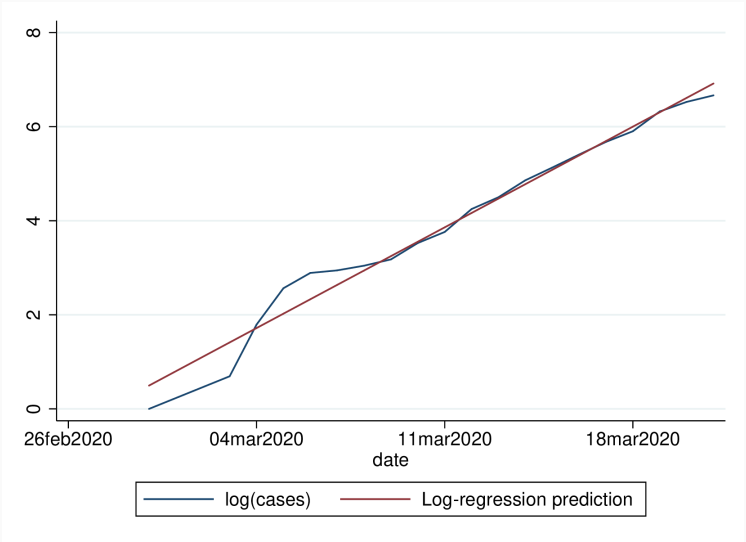
Stata output

```
. reg lc date
```

Source	SS	df	MS	Number of obs	=	20
				F(1, 18)	=	746.82
Model	66.1088015	1	66.1088015	Prob > F	=	0.0000
Residual	1.59336573	18	.088520318	R-squared	=	0.9765
				Adj R-squared	=	0.9752
Total	67.7021673	19	3.56327196	Root MSE	=	.29752

lc	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
date	.3058309	.0111911	27.33	0.000	.2823193	.3293426
_cons	-6719.833	246.0411	-27.31	0.000	-7236.746	-6202.92

Logs with log regression



Steady increase

The log of `cases` rises by 0.3058 per day

This means `cases` rises by a factor of $e^{0.3058} = 1.358$

The increase is 1.358 - 1 = 0.358, or almost 36% per day

Implies a doubling about every 2.6 days

But exponential increase is temporary

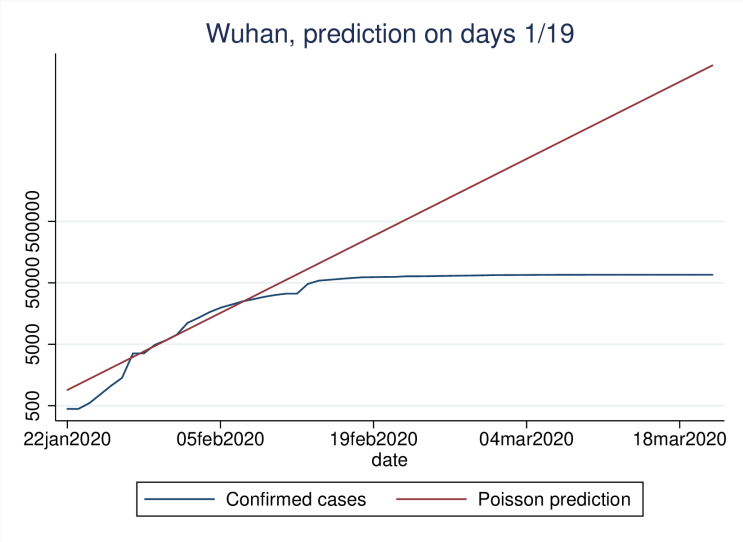
Exponential increase cannot go on indefinitely

Even if nothing is done, the rate of increase will decline as fewer people are left unexposed

And interventions (isolation, tracing) will reduce the rate

See China, for example

Wuhan, with prediction based on 1st 19 days



Summary

If there is a constant rate of increase, logs give us straight lines

Graph the log, or use a log scale on the Y-axis

Log regression allows us to estimate the rate

Exponential increase isn't forever, but modelling the exponential helps us see where the rate starts to drop

Code available here: http://teaching.sociology.ul.ie/so5032/irecovid.do

Outline

Today we introduce logistic regression: for binary outcomes

See Agresti Ch 15 Sec 1.

Binary outcomes and regression

- OLS (linear regression) requires an interval dependent variable
- Binary or “yes/no” dependent variables are not suitable
- Nor are rates, e.g., n successes out of m trials

Problems with OLS

- Errors are distinctly not normal
- While predicted value can be read as a probability, can depart from 0:1 range
- Particular difficulties with multiple explanatory variables
- Nonetheless still often used

Linear Probability Model

- If we use OLS with binary outcomes, it is called “linear probability model”:

$$Pr(Y = 1) = a + bX$$

- data is 0/1, prediction is probability
- Assumptions violated, but if predicted probabilities in range 0.2–0.8, not too bad

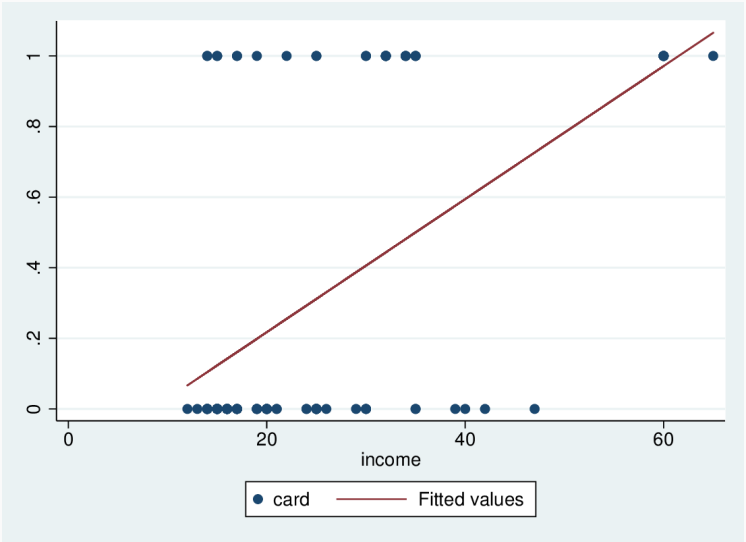
Credit card example

```
. reg card income
```

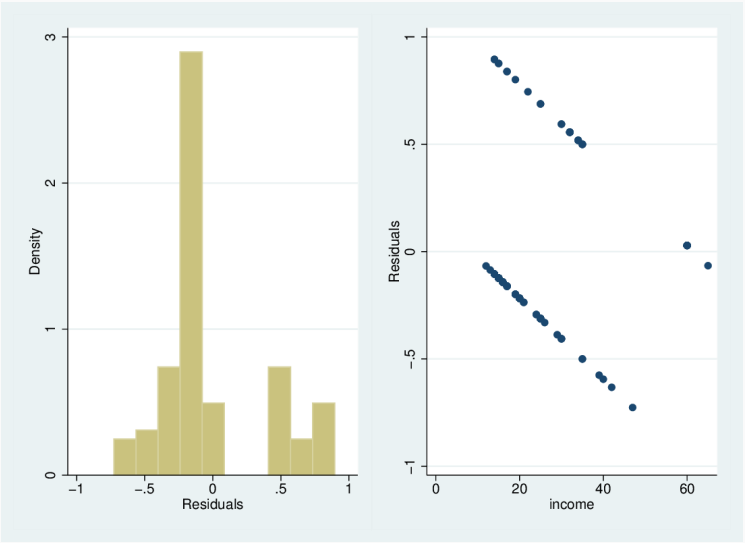
Source	SS	df	MS		
Model	5.55556122	1	5.55556122	Number of obs	= 100
Residual	15.8344388	98	.161575906	F(1, 98)	= 34.38
				Prob > F	= 0.0000
Total	21.39	99	.216060606	R-squared	= 0.2597
				Adj R-squared	= 0.2522
				Root MSE	= .40197

card	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
income	.0188458	.003214	5.86	0.000	.0124678	.0252238
_cons	-.1594495	.089584	-1.78	0.078	-.3372261	.018327

Credit card example



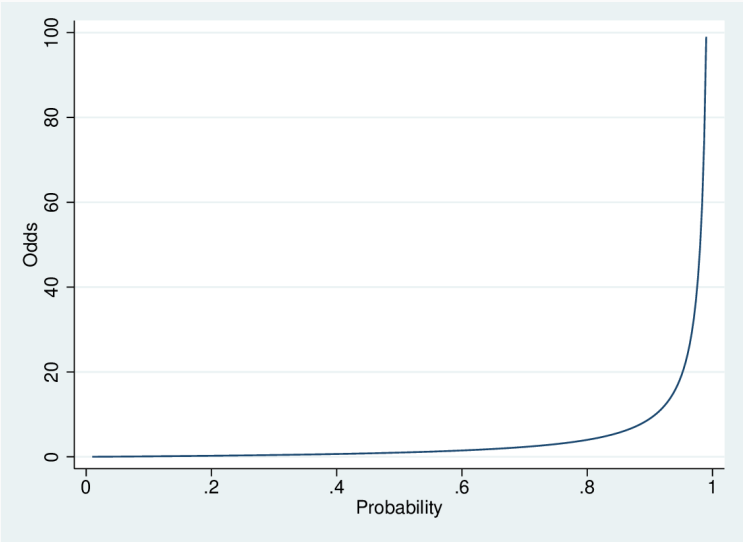
Credit card example



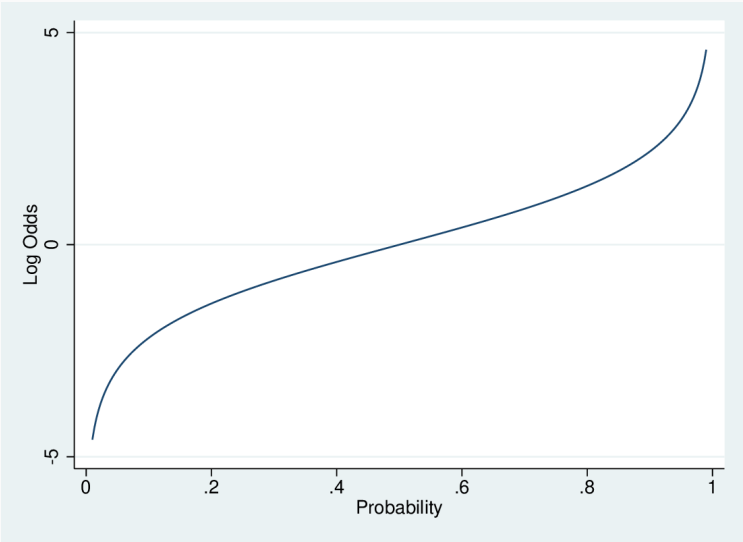
Logistic transformation

- Probability is bounded $[0 : 1]$
- OLS predicted value is unbounded
- How to transform probability to $-\infty : \infty$ range?
- Odds: $\frac{p}{1-p}$ – range is $0 : \infty$
- Log of odds: $\log \frac{p}{1-p}$ has range $-\infty : \infty$

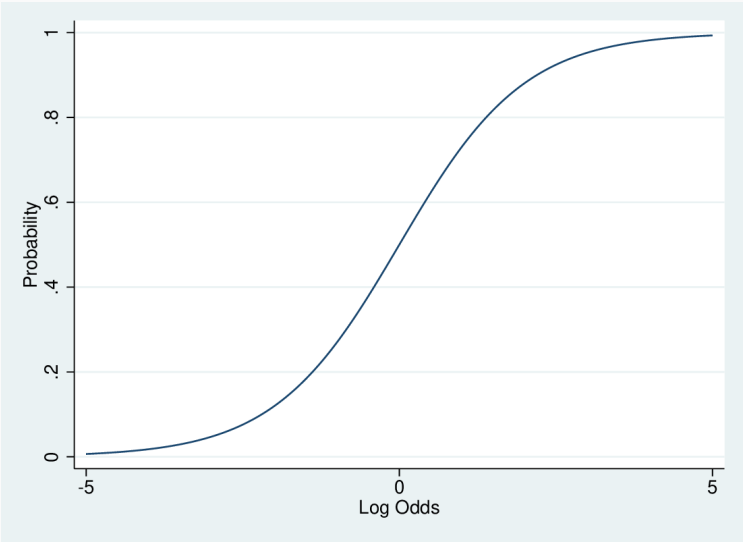
Probability to odds



Probability to log-odds



Rotated: the "S-shaped" curve



Logistic regression

- Logistic regression uses this as the dependent variable:

$$\log\left(\frac{p}{1-p}\right) = a + bX$$

Alternatives

We can look at this in three ways

- In terms of log-odds:

$$\log\left(\frac{Pr(Y=1)}{1-Pr(Y=1)}\right) = a + bX$$

- In terms of odds:

$$\frac{Pr(Y=1)}{1-Pr(Y=1)} = e^{a+bX}$$

- In terms of probability:

$$Pr(Y=1) = \frac{e^{a+bX}}{1+e^{a+bX}} = \frac{1}{1+e^{-a-bX}}$$

Parameters

- The b parameter is the effect of a unit change in X on $\log\left(\frac{Pr(Y=1)}{1-Pr(Y=1)}\right)$
- This implies a multiplicative change of e^b in $\frac{Pr(Y=1)}{1-Pr(Y=1)}$, in the Odds
- Thus an odds ratio
- But the effect of b on P depends on the level of b

Credit card logistic regression

```
. logit card income

Iteration 0:   log likelihood = -61.910066
Iteration 1:   log likelihood = -48.707265
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Iteration 3:   log likelihood =  -48.61304
Iteration 4:   log likelihood =  -48.61304

Logistic regression                             Number of obs     =        100
                                                LR chi2(1)        =      26.59
                                                Prob > chi2       =     0.0000
Log likelihood =  -48.61304                     Pseudo R2         =     0.2148

------------------------------------------------------------------------------
        card |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   .1054089   .0261574     4.03   0.000     .0541413    .1566765
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------------------------------------------------------------------------------
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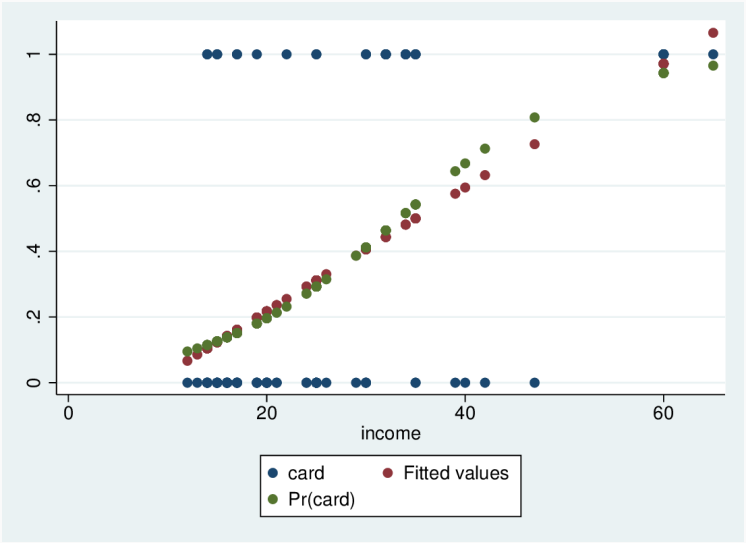

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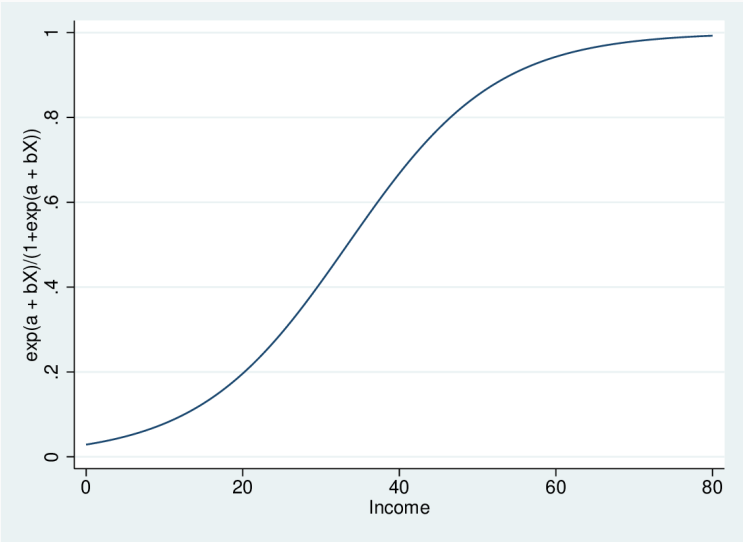

Calculating predicted probabilities by hand

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- First, plug them into the a + bX part to get the predicted log-odds
- Then take the anti-log of the log-odds to get the odds
- Then odds/(1+odds) gives us the probability

Calculating predicted probabilities

- Example: log(odds) = 0.25 + 0.12X
- Predict for X == 10
  - Predicted log-odds = 0.25 + 0.12*10 = 1.45
  - Predicted odds = $e^{1.45}$ = 4.263
  - Predicted probability = 4.263/(1 + 4.263) = 0.810

Web applet for practicing

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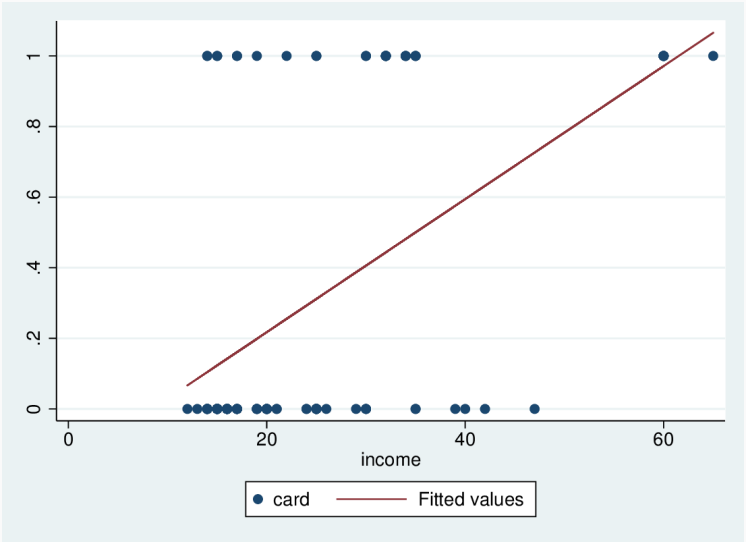

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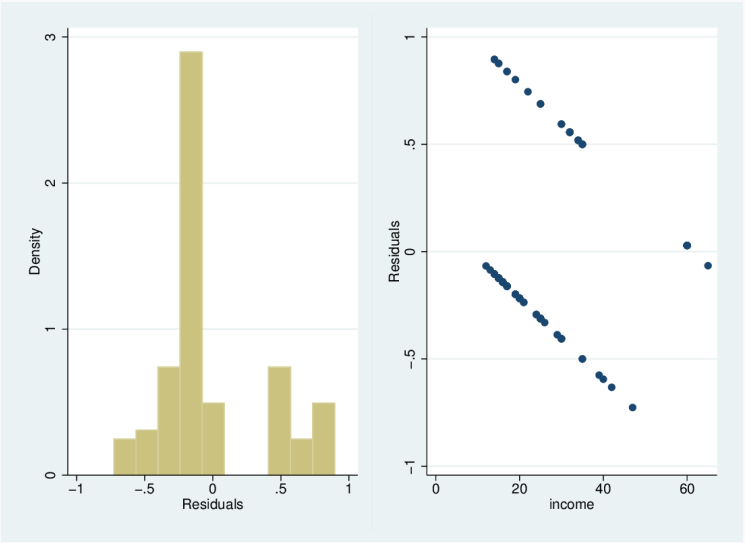

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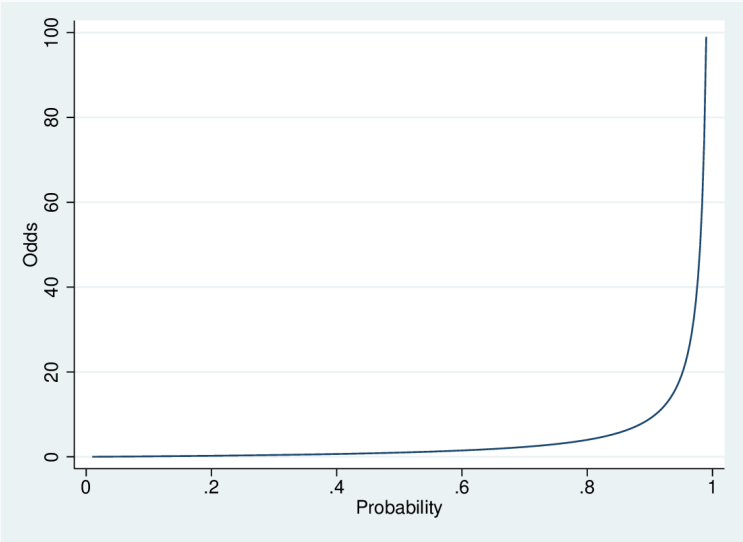
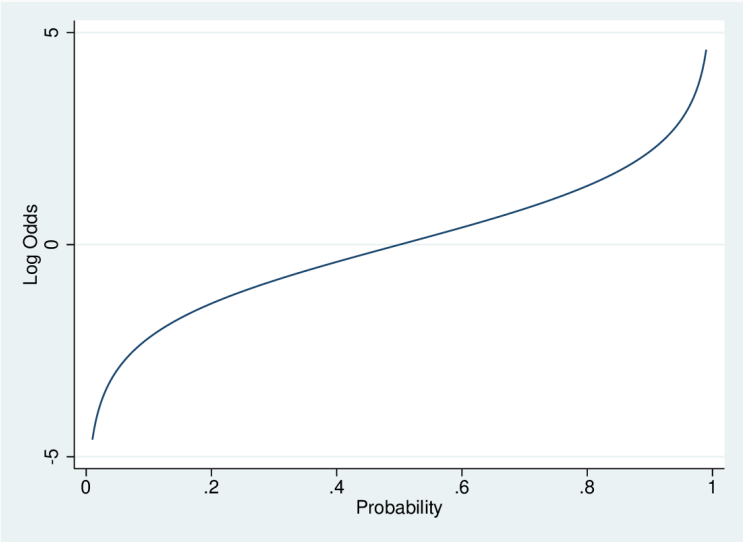


Probability to log-odds



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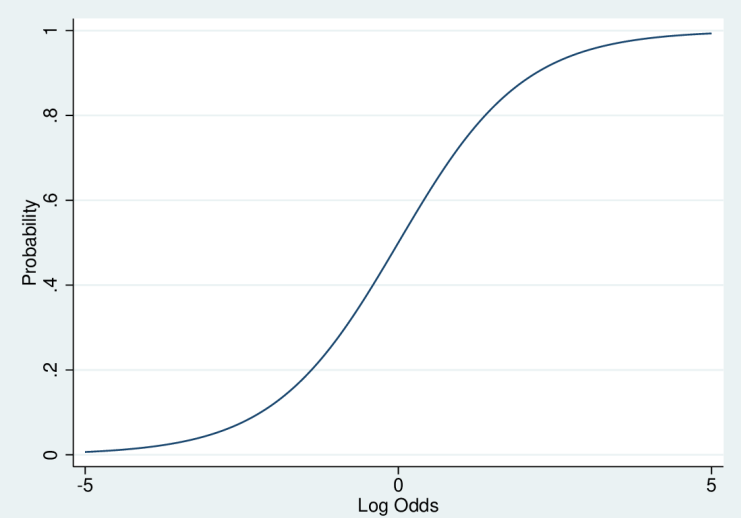

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------------------------------------------------------------------------------
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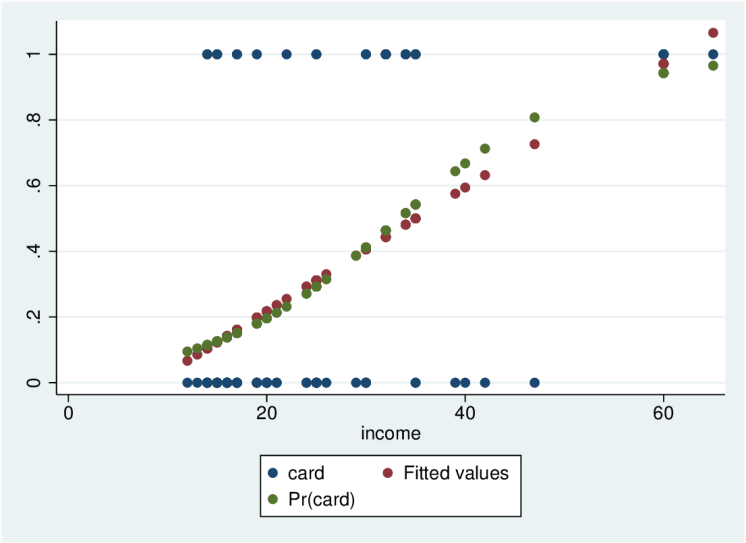

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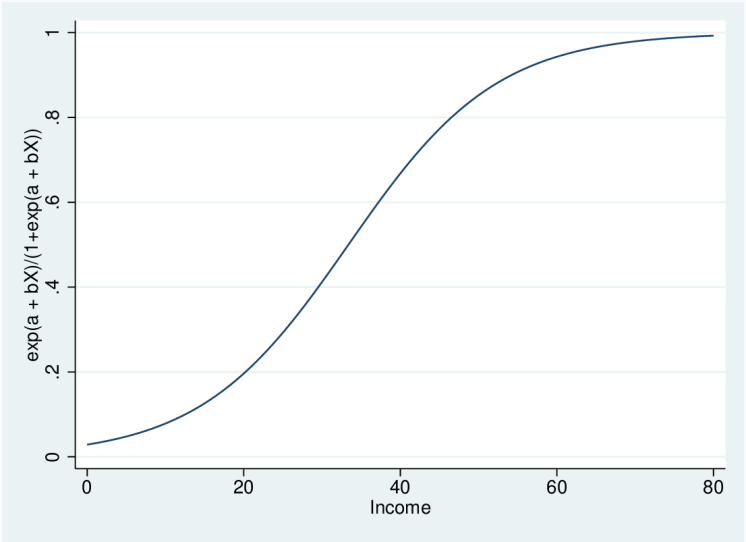

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Web applet for practicing

http://teaching.sociology.ul.ie:3838/logabx/

Housing tenure

- Housing tenure: probability of owning outright, BHPS data

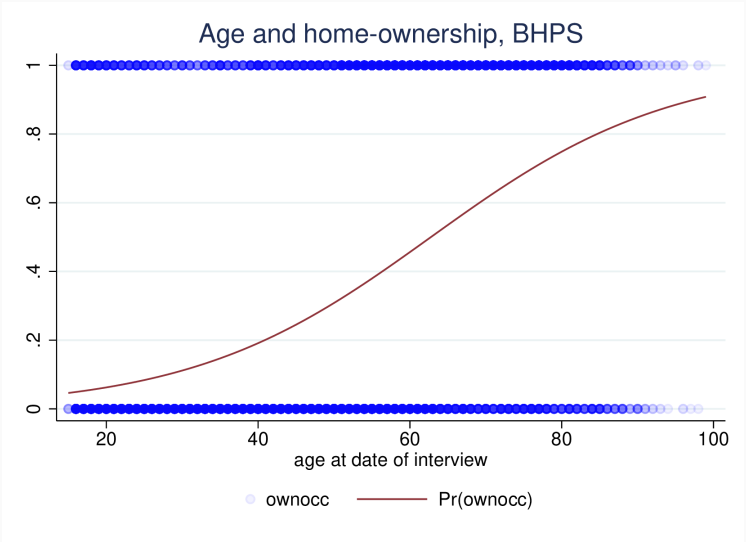
```
. logit ownocc age

Iteration 0:  Log likelihood = -8728.6773
Iteration 1:  Log likelihood = -7150.2389
Iteration 2:  Log likelihood = -7095.7194
Iteration 3:  Log likelihood = -7095.5268
Iteration 4:  Log likelihood = -7095.5268

Logistic regression                                     Number of obs =  14,182
                                                        LR chi2(1)    = 3266.30
                                                        Prob > chi2   =  0.0000
Log likelihood = -7095.5268                             Pseudo R2     =  0.1871

------------------------------------------------------------------------------
      ownocc | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         age |   .0633183   .0012705    49.84   0.000     .0608281    .0658084
       _cons |  -3.974023   .0697795   -56.95   0.000    -4.110788   -3.837258
------------------------------------------------------------------------------
```

Predictions



Predictions

$LO = a + bX$

$Odds = \exp(a + bX)$

$P = Odds/(1 + Odds)$

X increases by 1:

- LO by b (additive)
- Odds by e^b (multiplicative)
- P is more complicated

Predicton

- Log-odds

X = x	LO(x) = a + bx
X = x+1	LO(x+1) = a + b(x + 1) = a + bx + b
Difference:	LO(x+1) - LO(x) = b

Prediction: odds scale

- Odds

X = x	Odds(x) = $e^{a+bx} = e^a e^{bx}$
X = x+1	Odds(x+1) = $e^{a+b(x+1)} = e^{a+bx+b} = e^a e^{bx} e^b$
Ratio	Odds(x+1)/Odds(x) = e^b

- Hence odds-ratio: if X increases by 1, OR increases by factor of e^b

Odds ratio

```
. tab univ ownocc
```

univ	ownocc 0	1	Total
0	8,335	3,835	12,170
1	1,514	499	2,013
Total	9,849	4,334	14,183

OR = (499/1514) / (3835/8335) = 0.7163

```
. logit ownocc i.univ

Iteration 0:  Log likelihood =  -8729.863
Iteration 1:  Log likelihood = -8710.9025
Iteration 2:  Log likelihood = -8710.8468
Iteration 3:  Log likelihood = -8710.8468

Logistic regression                                     Number of obs = 14,183
                                                        LR chi2(1)    =  38.03
                                                        Prob > chi2   = 0.0000
Log likelihood = -8710.8468                             Pseudo R2     = 0.0022

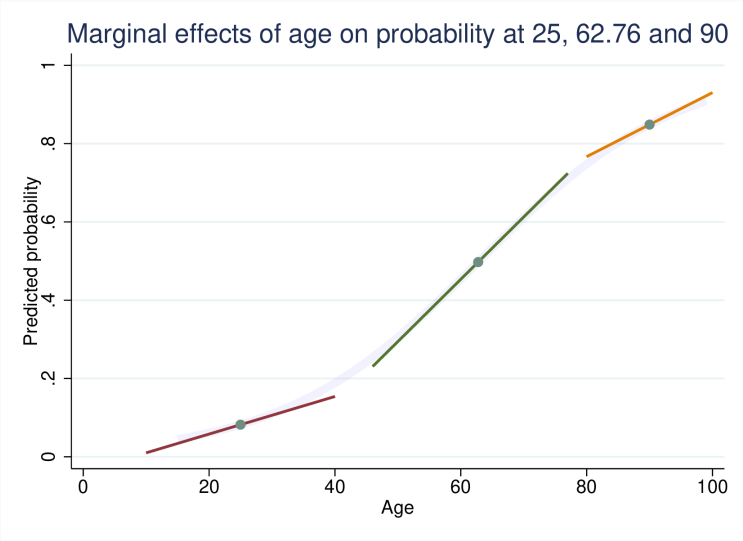
------------------------------------------------------------------------------
      ownocc | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
      1.univ |  -.3336103   .0551837    -6.05   0.000    -.4417683   -.2254522
       _cons |  -.7762941   .0195124   -39.78   0.000    -.8145376   -.7380506
------------------------------------------------------------------------------
```

$e^b = e^{-.3336103} = 0.7163$

Predictions on probability scale

- Effect of X on the probability scale is non-linear
- Low when p is either high or low
- Highest at p = 0.5, odds = 1, log-odds = 0
- The steepest slope is at p = 0.5, with a value of $\frac{\beta}{4}$

Marginal effects



Multiple explanatory variables

```
. logit owncc age i.univ

Iteration 0:  Log likelihood = -8728.6773
Iteration 1:  Log likelihood = -7150.3435
Iteration 2:  Log likelihood = -7094.4048
Iteration 3:  Log likelihood = -7094.1883
Iteration 4:  Log likelihood = -7094.1882

Logistic regression                                     Number of obs =  14,182
                                                        LR chi2(2)    = 3268.98
                                                        Prob > chi2   =  0.0000
Log likelihood = -7094.1882                             Pseudo R2     =  0.1873
```

owncc	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
age	.0636471	.0012888	49.38	0.000	.061121	.0661731
1.univ	.0999785	.0608614	1.64	0.100	-.0193076	.2192646
_cons	-4.004807	.0724889	-55.25	0.000	-4.146883	-3.862731

Lecture 11: Multinomial and Ordinal regression

Inference

Inference

- In practice, inference is similar to OLS though based on a different logic
- For each explanatory variable, $H_0 : \beta = 0$ is the interesting null
- $z = \frac{\hat{\beta}}{SE}$ is approximately normally distributed (large sample property)
- More usually, the Wald test is used: $\left(\frac{\hat{\beta}}{SE}\right)^2$ has a χ^2 distribution with one degree of freedom

Likelihood ratio tests

- The “likelihood ratio” test is thought more robust than the Wald test for smaller samples
- Where l_0 is the likelihood of the model without X_j, and l_1 that with it, the quantity

$$-2\left(\log\frac{l_0}{l_1}\right) = -2\left(\log l_0 - \log l_1\right)$$

is χ^2 distributed with one degree of freedom

Nested models

- More generally, $-2\left(\log \frac{l_0}{l_1}\right)$ tests nested models: where model 1 contains all the variables in model 0, plus m extra ones, it tests the null that all the extra β coefficients are zero (χ^2 with m df)
- If we compare a model against the null model (no explanatory variables, it tests

$$H_0 : \beta_1 = \beta_2 = \ldots = \beta_k = 0$$

- Strong analogy with F test in OLS

Example

```
. qui logit ownocc age

. est store mod1

. logit ownocc age i.educ

Iteration 0:  Log likelihood = -8728.6773
Iteration 1:  Log likelihood = -7136.2054
Iteration 2:  Log likelihood = -7077.7722
Iteration 3:  Log likelihood = -7077.5203
Iteration 4:  Log likelihood = -7077.5203

Logistic regression                                     Number of obs =  14,182
                                                        LR chi2(3)    = 3302.31
                                                        Prob > chi2   =  0.0000
Log likelihood = -7077.5203                             Pseudo R2     =  0.1892

------------------------------------------------------------------------------
      ownocc | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
         age |   .0652599   .0013433    48.58   0.000     .0626271    .0678927
             |
        educ |
        Med |   .3041599   .0673504     4.52   0.000     .1721556    .4361642
         Lo |  -.1075582   .0461399    -2.33   0.020    -.1979907   -.0171257
             |
       _cons |  -4.060514   .0730524   -55.58   0.000    -4.203694   -3.917333
------------------------------------------------------------------------------

. lrtest mod1

Likelihood-ratio test
Assumption: mod1 nested within .

LR chi2(2) =  36.01
Prob > chi2 =  0.0000
```

Lecture 11: Multinomial and Ordinal regression

Margins command

"Average Marginal Effect"

- "What would happen to the averege predicted probability if we increased X?"
- For linear regression, increase X by 1 => increase by b
  - increase X by 10 => increase by b× 10
  - increase X by 0.1 => increase by b× 0.1
  - since it's a straight line
- For AME in logistic we use the slope of the tangent, for each X value
- Average across the observed data
- Gives something like a LPM slope

AME in Stata

```
. margins, dydx(age)

Average marginal effects                                  Number of obs = 14,182
Model VCE: OIM

Expression: Pr(ownocc), predict()
dy/dx wrt:  age
```

	dy/dx	Delta-method std. err.	z	P>|z|	[95% conf.	interval]
age	.0104836	.0001382	75.84	0.000	.0102126	.0107545

Lecture 11: Multinomial and Ordinal regression

Maximum likelihood

Maximum likelihood estimation

- What is this “likelihood”?
- Unlike OLS, logistic regression (and many, many other models) are extimated by *maximum likelihood estimation*
- In general this works by choosing values for the parameter estimates which maximise the probability (likelihood) of observing the actual data
- OLS can be ML estimated, and yields exactly the same results

Iterative search

- Sometimes the values can be chosen analytically
  - A likelihood function is written, defining the probability of observing the actual data given parameter estimates
  - Differential calculus derives the values of the parameters that maximise the likelihood, for a given data set
- Often, such “closed form solutions” are not possible, and the values for the parameters are chosen by a systematic computerised search (multiple iterations)
- Extremely flexible, allows estimation of a vast range of complex models within a single framework

Likelihood as a quantity

- Either way, a given model yields a specific maximum likelihood for a give data set
- This is a probability, henced bounded [0 : 1]
- Reported as log-likelihood, hence bounded $[-\infty : 0]$
- Thus is usually a large negative number
- Where an iterative solution is used, likelihood at each stage is usually reported – *normally* getting nearer 0 at each step

Lecture 11: Multinomial and Ordinal regression

Tabular data

Tabular data

- If all the explanatory variables are categorical (or have few fixed values) your data set can be represented as a table
- If we think of it as a table where each cell contains n yeses and $m - n$ noes (n successes out of m trials) we can fit grouped logistic regression
- n successes out of m trials implies a binomial distribution of degree m

$$\log \frac{n}{m-n} = \alpha + \beta X$$

- The parameter estimates will be exactly the same as if the data were treated individually

Tabular data and goodness of fit

- But unlike with individual data, we can calculate goodness of fit, by relating observed successes to predicted in each cell
- If these are close we cannot reject the null hypothesis that the model is incorrect (i.e., you want a high p-value)
- Where l_i is the likelihood of the current model, and l_s is the likelihood of the "saturated model" the test statistic is

$$-2\left(\log\frac{l_i}{l_s}\right)$$

- The saturated model predicts perfectly and has as many parameters as there are "settings" (cells in the table)
- The test has *df* of number of settings less number of parameters estimated, and is χ^2 distributed