# SO5032 Lecture 3

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# Outline

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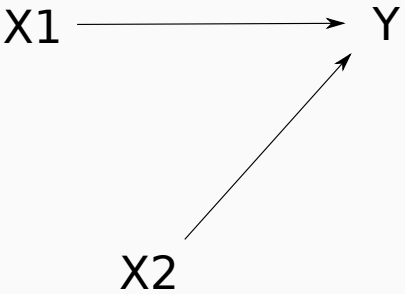
## Multidimensional causality

# Multidimensional causality

- Regression analysis never proves causal relationships, but it "thinks" in causal terms
- To use it we need to understand causal relationships: what process generates the data we see, and what can regression tell us about it.
- Start by considering the relationship between variables and patterns of association

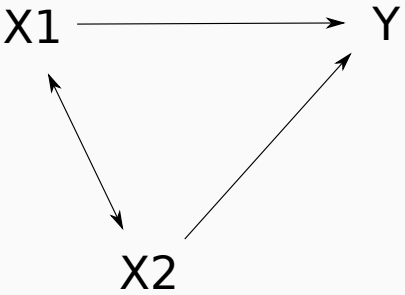
# 3-variable pictures

- Let's consider patterns of causality and association between three variables, X1 and X2, and Y
- If X1 and X2 are not correlated with each other, their separate effects on Y more or less just add up



# Correlated X variables

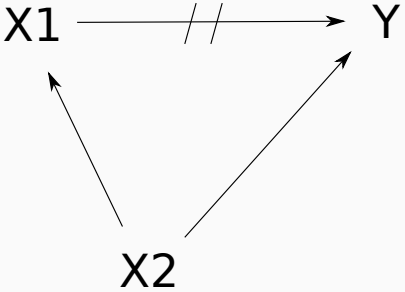
- But if X1 and X2 are correlated, things can get funny:




- In particular, if we measure the effect of one X without taking account of the other we will likely over-estimate it

# Spurious association

- X1 may have an association with Y, implying a causal relationship
- But if X2 affects both X1 and Y the relationship between X1 and Y may be spurious



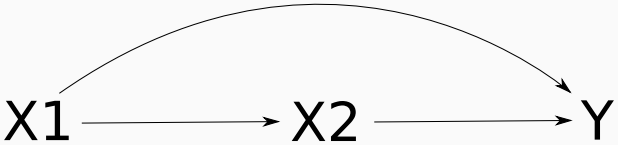
# Indirect effects

- Where there is a time-order (X1 before X2), we may see direct and indirect effects
- X1 may affect X2, which affects Y, but not affect Y directly
- Thus there is association between X1 and Y without a direct causal effect

$$X1 \longrightarrow X2 \longrightarrow Y$$

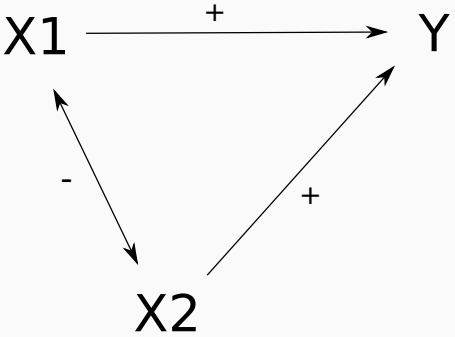
# Direct and indirect effects

- However, it is possible for both direct and indirect effects to be present at the same time



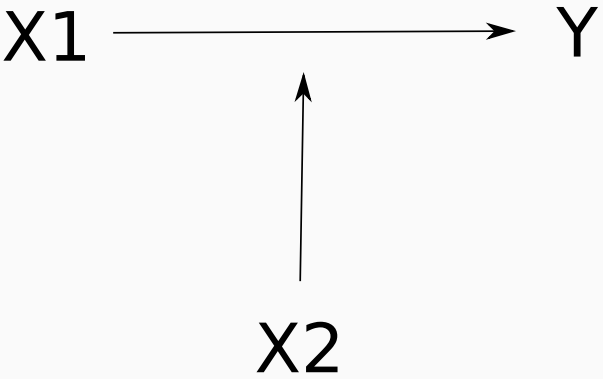
# Suppression

- Where X1 and X2 have positive effects on Y, but a negative correlation, or different effects on Y with a positive correlation, the association between X1 and Y may be suppressed
- That is, it may be invisible if we don't take account of X2



# Interactions

- An interaction effect is where the effect of one variable on Y changes depending on the value of another



# SO5032 Lecture 3

## Multiple regression

# Multiple explanatory variables

- Regression analysis can be extended to the case where there is more than one explanatory variable – multivariate regression
- This allows us to estimate the net simultaneous effect of many variables, and thus to begin to disentangle more complex relationships
- Interpretation is relatively easy: each variable gets its own slope coefficient, standard error and significance
- The slope coefficient is the effect on the dependent variable of a 1 unit change in the explanatory variable, *while taking account of the other variables*

# Example

- Example: income may be affected by gender, and also by paid work time: competing explanations – one or the other, or both could have effects
- We can fit bivariate regressions:

$$Income = a + b \times PaidWork$$

or

$$Income = a + b \times Female$$

- We can also fit a single multivariate regression

$$Income = a + b \times PaidWork + c \times Female$$

# Dichotomous variables

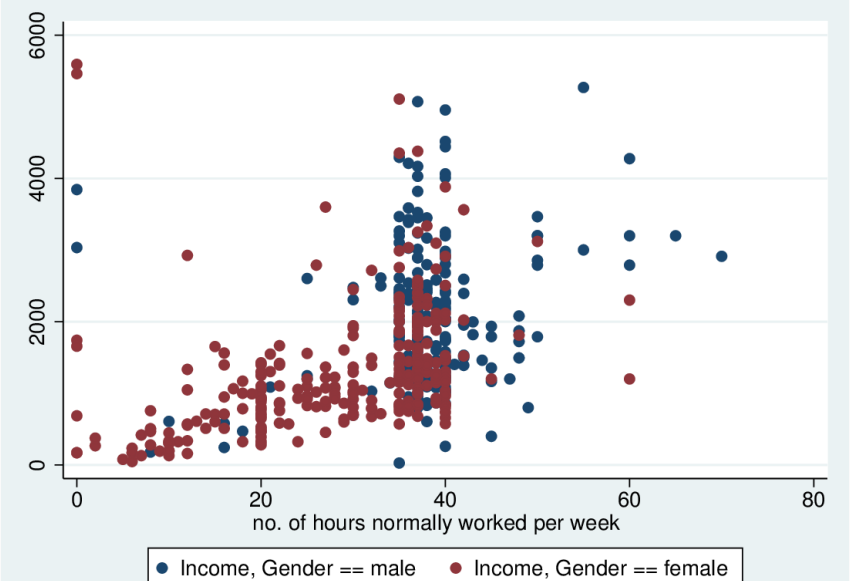
- We deal with gender in a special way: this is a *binary* or *dichotomous* variable – has two values
- We turn it into a yes/no or 0/1 variable – *e.g.*, female or not
- If we put this in as an explanatory variable a *one-unit change in the explanatory variable* is the difference between being male and female
- Thus the $c$ coefficient we get in the $Income = a + b \times PaidWork + c \times Female$ regression is the net change in predicted income for females, once you take account of paid work time.
- The $b$ coefficient is then the net effect of a unit change in paid work time, once you take gender into account.

# Income, hours and gender

```
. corr Income Gender Hours
(obs=506)
```

| | Income | Gender | Hours |
|---|---|---|---|
| Income | 1.0000 | | |
| Gender | -0.3280 | 1.0000 | |
| Hours | 0.3638 | -0.4360 | 1.0000 |

# Income, hours and gender



# T-test: Income by gender

```
. ttest Income, by(Gender)

Two-sample t test with equal variances
```

| Group | Obs | Mean | Std. Err. | Std. Dev. | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| male | 437 | 1618.348 | 59.11677 | 1235.809 | 1502.159 | 1734.537 |
| female | 531 | 992.1805 | 40.82127 | 940.6625 | 911.9892 | 1072.372 |
| combined | 968 | 1274.861 | 36.23219 | 1127.281 | 1203.759 | 1345.964 |
| diff | | 626.1674 | 70.00484 | | 488.7883 | 763.5465 |

```
    diff = mean(male) - mean(female)                              t =   8.9446
Ho: diff = 0                                     degrees of freedom =      966

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000
```

# Regression: Just hours

```
. reg Income Hours

      Source |       SS           df       MS      Number of obs   =       506
-------------+----------------------------------   F(1, 504)       =     76.86
       Model |  86947928.8         1  86947928.8   Prob > F        =    0.0000
    Residual |   570128215       504  1131206.78   R-squared       =    0.1323
-------------+----------------------------------   Adj R-squared   =    0.1306
       Total |   657076144       505  1301140.88   Root MSE        =    1063.6

------------------------------------------------------------------------------
      Income |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       Hours |   37.82204   4.314061     8.77   0.000     29.34628     46.2978
       _cons |   449.7435   150.1722     2.99   0.003      154.703    744.7841
------------------------------------------------------------------------------
```

# Regression: Hours and binary gender

```
. reg Income Hours i.Gender

      Source |       SS           df       MS      Number of obs   =       506
-------------+----------------------------------   F(2, 503)       =     50.70
       Model |  110236231         2  55118115.6   Prob > F        =    0.0000
    Residual |  546839912       503  1087156.88   R-squared       =    0.1678
-------------+----------------------------------   Adj R-squared   =    0.1645
       Total |  657076144       505  1301140.88   Root MSE        =    1042.7

------------------------------------------------------------------------------
      Income |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       Hours |   28.33857   4.699451     6.03   0.000      19.1056    37.57155
             |
      Gender |
     female  |  -478.4214   103.3684    -4.63   0.000    -681.5084   -275.3344
       _cons |   1022.139   192.2717     5.32   0.000     644.3844    1399.893
------------------------------------------------------------------------------
```

# Regression: for men only

```
. reg Income Hours if Gender==1

      Source |       SS           df       MS      Number of obs   =       232
-------------+----------------------------------   F(1, 230)       =      5.36
       Model |  8009519.02         1  8009519.02   Prob > F        =    0.0215
    Residual |   343845612       230  1494980.92   R-squared       =    0.0228
-------------+----------------------------------   Adj R-squared   =    0.0185
       Total |   351855131       231  1523182.38   Root MSE        =    1222.7

------------------------------------------------------------------------------
      Income |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       Hours |   24.61855   10.63597     2.31   0.022     3.662162    45.57495
       _cons |   1164.366   414.4901     2.81   0.005     347.6826    1981.049
------------------------------------------------------------------------------
```

# Regression: for women only

```
. reg Income Hours if Gender==2

      Source |       SS           df       MS      Number of obs   =       274
-------------+----------------------------------   F(1, 272)       =     42.63
       Model |  31772944.2         1  31772944.2   Prob > F        =    0.0000
    Residual |   202744304       272  745383.469   R-squared       =    0.1355
-------------+----------------------------------   Adj R-squared   =    0.1323
       Total |   234517248       273  859037.537   Root MSE        =    863.36

------------------------------------------------------------------------------
      Income |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       Hours |   29.70376   4.549594     6.53   0.000     20.74687    38.66065
       _cons |   504.6153   140.3614     3.60   0.000     228.2824    780.9482
------------------------------------------------------------------------------
```

# Regression: interaction

```
. reg Income c.Hours##i.Gender

      Source |       SS           df       MS      Number of obs   =       506
-------------+----------------------------------   F(3, 502)       =     33.82
       Model |  110486228         3  36828742.8   Prob > F        =    0.0000
    Residual |  546589915       502  1088824.53   R-squared       =    0.1681
-------------+----------------------------------   Adj R-squared   =    0.1632
       Total |  657076144       505  1301140.88   Root MSE        =    1043.5

------------------------------------------------------------------------------
      Income |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       Hours |   24.61855   9.076915     2.71   0.007     6.785132    42.45198
             |
      Gender |
     female  |  -659.7502   392.3082    -1.68   0.093    -1430.518    111.0181
             |
Gender#c.Hours |
     female  |   5.085207   10.61255     0.48   0.632    -15.76529     25.9357
             |
       _cons |   1164.366   353.7327     3.29   0.001     469.3865    1859.345
------------------------------------------------------------------------------
```

# Regression: Direct and indirect 1

```
. reg ownscore fatherscore

      Source |       SS           df       MS      Number of obs   =     1,000
-------------+----------------------------------   F(1, 998)       =     53.50
       Model |  13269.3853         1  13269.3853   Prob > F        =    0.0000
    Residual |  247525.861       998  248.021905   R-squared       =    0.0509
-------------+----------------------------------   Adj R-squared   =    0.0499
       Total |  260795.247       999  261.056303   Root MSE        =    15.749

------------------------------------------------------------------------------
    ownscore |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 fatherscore |   .2370829    .032413     7.31   0.000     .1734773    .3006884
       _cons |   37.90861   1.672157    22.67   0.000     34.62726    41.18996
------------------------------------------------------------------------------
```

# Regression: Direct and indirect 2

```
. reg education fatherscore

      Source |       SS           df       MS      Number of obs   =     1,000
-------------+----------------------------------   F(1, 998)       =    111.01
       Model |  311.104929         1  311.104929   Prob > F        =    0.0000
    Residual |  2797.00607       998  2.80261129   R-squared       =    0.1001
-------------+----------------------------------   Adj R-squared   =    0.0992
       Total |   3108.111       999  3.11122222   Root MSE        =    1.6741

------------------------------------------------------------------------------
   education |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 fatherscore |   .0363018   .0034455    10.54   0.000     .0295405    .0430631
       _cons |   1.295213   .1777516     7.29   0.000     .9464035    1.644023
------------------------------------------------------------------------------
```

# Regression: Direct and indirect 3

```
. reg ownscore education

      Source |       SS           df       MS      Number of obs   =     1,000
-------------+----------------------------------   F(1, 998)       =    447.54
       Model |  80742.8091         1  80742.8091   Prob > F        =    0.0000
    Residual |  180052.437       998  180.413264   R-squared       =    0.3096
-------------+----------------------------------   Adj R-squared   =    0.3089
       Total |  260795.247       999  261.056303   Root MSE        =    13.432

------------------------------------------------------------------------------
    ownscore |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   education |   5.096871   .2409273    21.16   0.000     4.624089    5.569653
       _cons |   33.87079   .8556481    39.58   0.000     32.19171    35.54986
------------------------------------------------------------------------------
```

# Regression: Direct and indirect 4

```
. reg ownscore education fatherscore

      Source |       SS           df       MS      Number of obs   =     1,000
-------------+----------------------------------   F(2, 997)       =    226.41
       Model |  81453.7212         2  40726.8606   Prob > F        =    0.0000
    Residual |  179341.525       997  179.881169   R-squared       =    0.3123
-------------+----------------------------------   Adj R-squared   =    0.3109
       Total |  260795.247       999  261.056303   Root MSE        =    13.412

------------------------------------------------------------------------------
    ownscore |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   education |   4.937369   .2535982    19.47   0.000     4.439722    5.435017
 fatherscore |   .0578475   .0290984     1.99   0.047     .0007463    .1149486
       _cons |   31.51367   1.461439    21.56   0.000     28.64582    34.38152
------------------------------------------------------------------------------
```