SO5041, Autumn 2016

Unit 16: Regression

Regression analysis

- Regression Analysis: Fitting the "best" line through the scatter
- Very closely related to correlation, but treats one variable as **dependent** and the other(s) as **explanatory**, while correlation is asymmetric

Some geometry: equation of a line

$$Y = a + bX$$

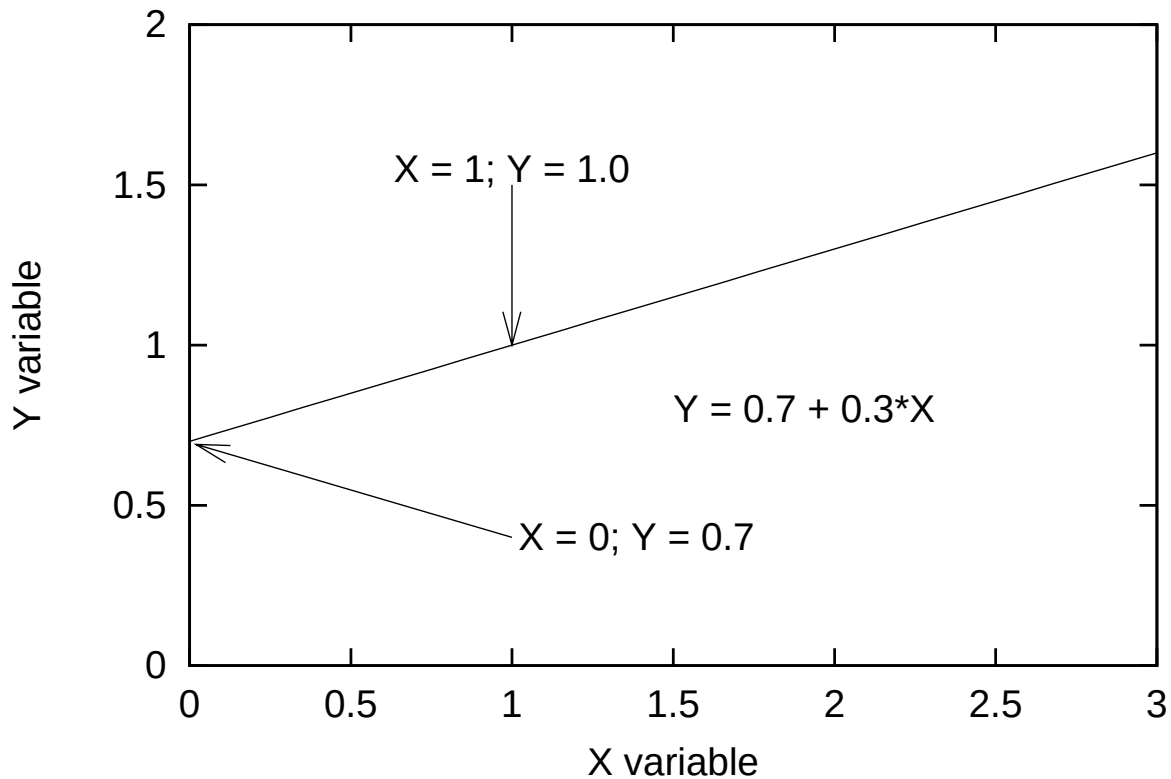


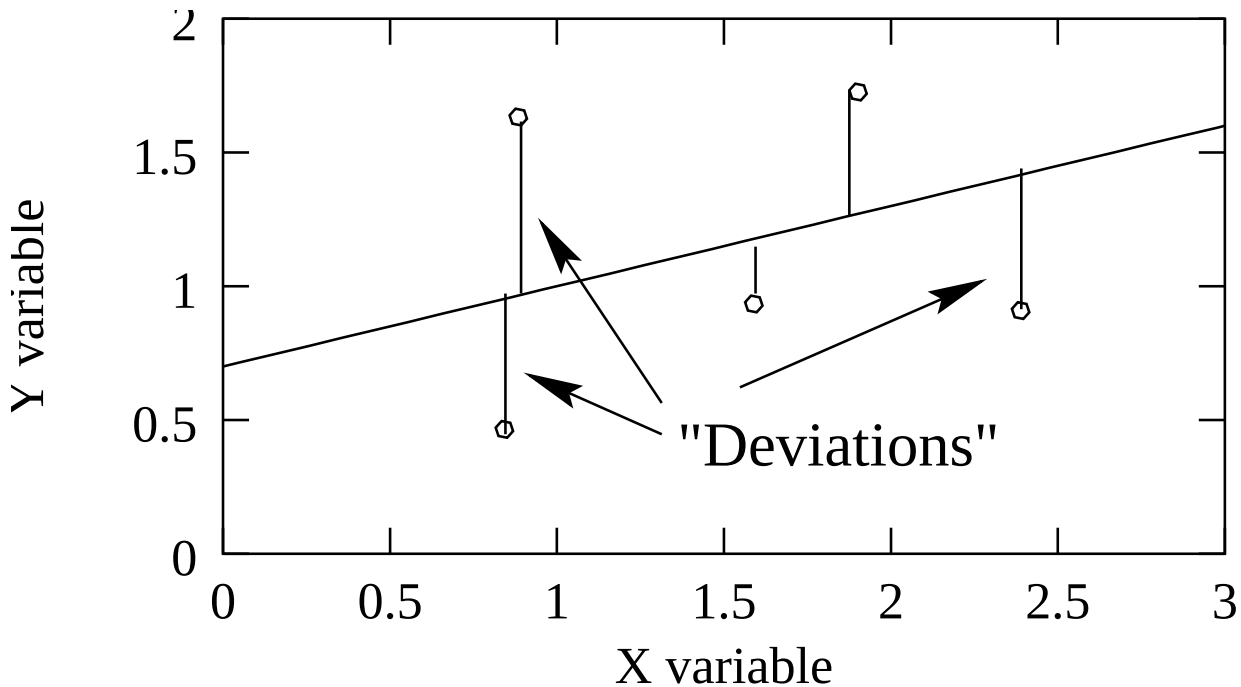
Figure 9: The equation of a line.

Predictive in intent

- Asymmetric: use X to predict Y
- PRE: implied causality
  (Proportional Reduction in Error: if we know X, we can guess Y better)
- Find the best *a* and *b* to summarise the data scatter:
  - 'Best' is defined as minimising the squared deviations between the observed data-points and the fitted line, hence often called 'least-squares' regression
  - Deviations are the vertical distance between the line and the observed data points.
  - Very similar logic to the mean (minimise variance).

Deviations again

Figure 10: Deviations from the line



Predicted values

- The line gives a **predicted** value of Y for each value of X:

$$\hat{Y} = a + bX$$

$$Y = \hat{Y} + e$$

$$Y = a + bX + e$$

  e is the 'residual' or deviation.
- That is, knowing X we "predict" or guess Y as $a + bX$
- In general this is more accurate that guessing Y as $\bar{Y}$, the mean: "Proportionate Reduction in Error"

Regression equation

- Regression equation: the estimate of Y, called $\hat{Y}$, depends on X:

$$\hat{Y} = a + bX$$

- The regression slope b depends on SXY and SXX, the intercept a is calculated from b and the mean values of Y and X:

$$b = \frac{SXY}{SXX}$$

$$a = \bar{Y} - b\bar{X}$$

$$SXX = \sum (X_i - \bar{X})^2$$

$$SXY = \sum (X_i - \bar{X})(Y_i - \bar{Y})$$

Pitfalls

- Like correlation, non-linear relationships may be missed
- Spurious relationships will fit just as well as real ones (*e.g.*, if A affects B and A affects C, B and C will seem to be related and a regression line might fit well)
- Predicting outside the range of the data: the relationship we see only holds for the data we use, and it may well not hold for higher (or lower) values of X and Y

Fit

- How well does it "fit"? We use R^2 to tell:
  - ranges from 0: no relationship at all
  - to 1: perfect relationship, all Ys are exactly equal to $a + bX$
  - values from 0.7 up indicate quite a good relationship
  - smaller values may indicate an interesting relationship
- In the case of bivariate regression (one independent variable), R^2 is the same as $r \times r$ (squared correlation coefficient).

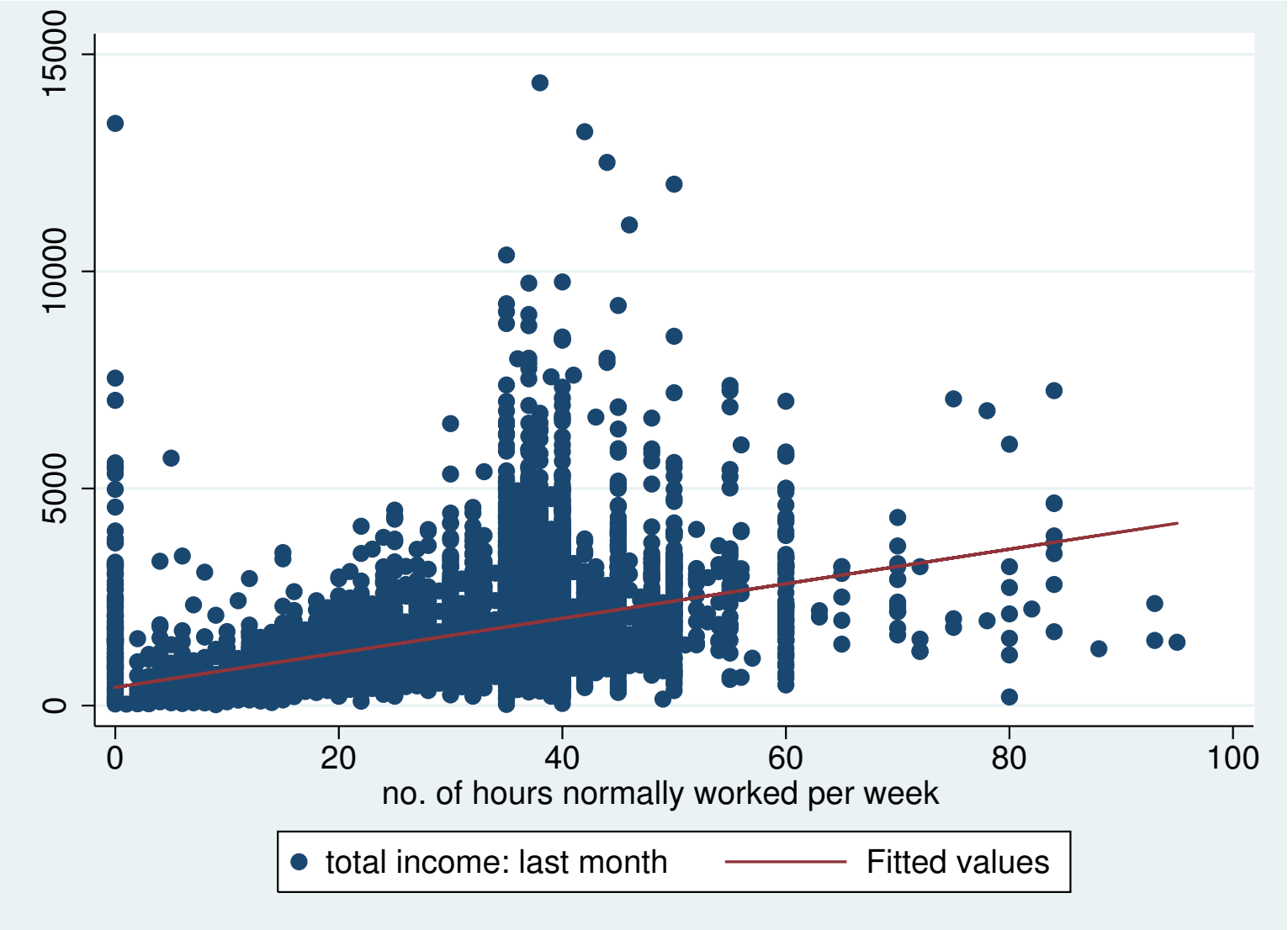
Regression in Stata:

```
. reg ofimn ojbhrs

      Source |       SS       df       MS              Number of obs =    7894
-------------+------------------------------           F(  1,  7892) = 1406.39
       Model |  1.7116e+09     1  1.7116e+09           Prob > F      =  0.0000
    Residual |  9.6048e+09  7892  1217033.86           R-squared     =  0.1513
-------------+------------------------------           Adj R-squared =  0.1511
       Total |  1.1316e+10  7893  1433733.83           Root MSE      =  1103.2

------------------------------------------------------------------------------
       ofimn |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      ojbhrs |   39.77896   1.060718    37.50   0.000     37.69967    41.85825
       _cons |   420.2139   37.02457    11.35   0.000     347.6359    492.7919
------------------------------------------------------------------------------
```

Predicted regression line



Multiple explanatory variables

- Regression analysis can be extended to the case where there is more than one explanatory variable – multivariate regression
- This allows us to estimate the net simultaneous effect of many variables, and thus to begin to disentangle more complex relationships
- Interpretation is relatively easy: each variable gets its own slope coefficient, standard error and significance
- The slope coefficient is the effect on the dependent variable of a 1 unit change in the explanatory variable, *while taking account of the other variables*

Example

- Example: domestic work time may be affected by gender, and also by paid work time: competing explanations – one or the other, or both could have effects
- We can fit bivariate regressions:

$$DWT = a + b \times PaidWork$$

or

$$DWT = a + b \times Female$$

- We can also fit a single multivariate regression

$$DWT = a + b \times PaidWork + c \times Female$$

Dichotomous variables

- We deal with gender in a special way: this is a *binary* or *dichotomous* variable – has two values
- We turn it into a yes/no or 0/1 variable – *e.g.*, female or not
- If we put this in as an explanatory variable a *one unit change in the explanatory variable* is the difference between being male and female
- Thus the c coefficient we get in the $DWT = a + b \times PaidWork + c \times Female$ regression is the net change in predicted domestic work time for females, once you take account of paid work time.
- The b coefficient is then the net effect of a unit change in paid work time, once you take gender into account.

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Unit 17: Review

- Retrospective Course Outline:
  - Representing information as numbers
  - Probability distributions and CIs
  - Hypothesis testing
  - Questionnaires and sampling
  - Correlation and regression

- Representing information as numbers
  - levels of measurement
  - univariate descriptive statistics and graphics
  - bivariate descriptive statistics and graphics
  - Agresti/Finlay ch 2.1, ch 3, assignment 1
- Probability distributions and CIs
  - The normal distribution
  - The t-distribution
  - The χ^2 (chi-sq) distribution
  - Confidence intervals
  - Agresti/Finlay chs 4, 5, assignment 2

- Hypothesis testing
  - Extension of CI framework
  - Test the "null hypothesis", if rejected ⇒support for initial hypothesis
  - Agresti/Finlay ch 6
- Questionnaires and sampling
  - Questionnaire design
  - The principles of sampling
  - Different sampling strategies
  - Sampling theory: Agresti/Finlay ch 2.2–2.4
  - Methodological overview: de Vaus, chs 6–8

- Correlation and regression
  - Correlation as bivariate summary of interval/ratio variables
  - Regression: $\hat{Y} = a + bX$; directional
  - Agresti ch 9.1–9.4