SO5041 Unit 16: Regression

Brendan Halpin, Sociology

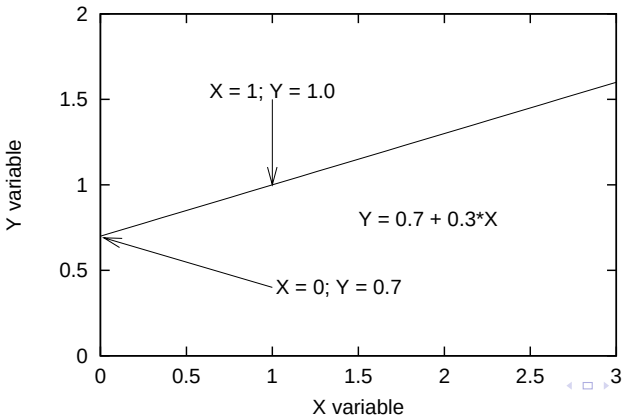
Autumn 2019/0

Regression analysis

- Regression Analysis: Fitting the "best" line through the scatter
- Very closely related to correlation, but treats one variable as dependent and the other(s) as explanatory, while correlation is asymmetric

Some geometry: equation of a line

$$Y = a + bX$$



Predictive in intent

- Asymmetric: use X to predict Y
- PRE: implied causality
  (Proportional Reduction in Error: if we know X, we can guess Y better)
- Find the best *a* and *b* to summarise the data scatter:
  - 'Best' is defined as minimising the squared deviations between the observed data-points and the fitted line, hence often called 'least-squares' regression
  - Deviations are the vertical distance between the line and the observed data points.
  - Very similar logic to the mean (minimise variance).

Predicted values

- The line gives a predicted value of Y for each value of X:

$$\hat{Y} = a + bX$$

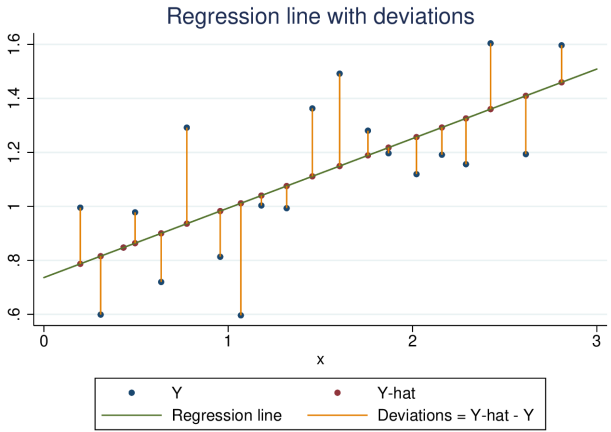
$$Y = \hat{Y} + e$$

$$Y = a + bX + e$$

  e is the 'residual' or deviation.

- That is, knowing X we "predict" or guess Y as $a + bX$
- In general this is more accurate that guessing Y as $\bar{Y}$, the mean: "Proportionate Reduction in Error"

Deviations from the line



Regression equation

- Regression equation: the estimate of Y, called $\hat{Y}$, depends on X:

$$\hat{Y} = a + bX$$

- The regression slope b depends on SXY and SXX, the intercept a is calculated from b and the mean values of Y and X:

$$b = \frac{SXY}{SXX}$$

$$a = \bar{Y} - b\bar{X}$$

$$SXX = \sum (X_i - \bar{X})^2$$

$$SXY = \sum (X_i - \bar{X})(Y_i - \bar{Y})$$

Pitfalls

- Like correlation, non-linear relationships may be missed
- Spurious relationships will fit just as well as real ones (e.g., if A affects B and A affects C, B and C will seem to be related and a regression line might fit well)
- Predicting outside the range of the data: the relationship we see only holds for the data we use, and it may well not hold for higher (or lower) values of X and Y

Fit

- How well does it "fit"? We use R^2 to tell:
  - ranges from 0: no relationship at all
  - to 1: perfect relationship, all Ys are exactly equal to $a + bX$
  - values from 0.7 up indicate quite a good relationship
  - smaller values may indicate an interesting relationship
- In the case of bivariate regression (one independent variable), R^2 is the same as $r \times r$ (squared correlation coefficient).

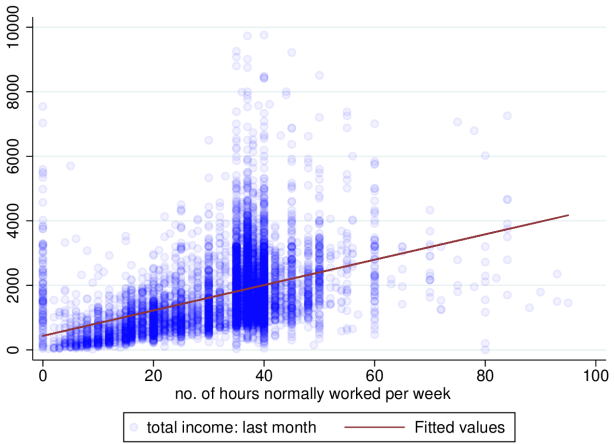
Regression in Stata:

```
. reg ofimn ojbhrs
```

Source	SS	df	MS		
				Number of obs	= 7,945
				F(1, 7943)	= 1398.95
Model	1.7000e+09	1	1.7000e+09	Prob > F	= 0.0000
Residual	9.6522e+09	7,943	1215179.2	R-squared	= 0.1497
				Adj R-squared	= 0.1496
Total	1.1352e+10	7,944	1429021.17	Root MSE	= 1102.4

ofimn	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
ojbhrs	39.34202	1.051854	37.40	0.000	37.28011	41.40393
_cons	434.7389	36.8029	11.81	0.000	362.5955	506.8822

Predicted regression line



Multiple explanatory variables

- Regression analysis can be extended to the case where there is more than one explanatory variable – multivariate regression
- This allows us to estimate the net simultaneous effect of many variables, and thus to begin to disentangle more complex relationships
- Interpretation is relatively easy: each variable gets its own slope coefficient, standard error and significance
- The slope coefficient is the effect on the dependent variable of a 1 unit change in the explanatory variable, *while taking account of the other variables*

Example

- Example: domestic work time may be affected by gender, and also by paid work time: competing explanations – one or the other, or both could have effects
- We can fit bivariate regressions:

$$DWT = a + b \times PaidWork$$

or

$$DWT = a + b \times Female$$

- We can also fit a single multivariate regression

$$DWT = a + b \times PaidWork + c \times Female$$

Dichotomous variables

- We deal with gender in a special way: this is a *binary* or *dichotomous* variable – has two values
- We turn it into a yes/no or 0/1 variable – e.g., female or not
- If we put this in as an explanatory variable a *one unit change in the explanatory variable* is the difference between being male and female
- Thus the c coefficient we get in the $DWT = a + b \times PaidWork + c \times Female$ regression is the net change in predicted domestic work time for females, once you take account of paid work time.
- The b coefficient is then the net effect of a unit change in paid work time, once you take gender into account.

Sex only predicting income

```
. reg ofimn i.osex
```

Source	SS	df	MS		
Model	805586626	1	805586626	Number of obs	= 7,945
Residual	1.0547e+10	7,943	1327780.12	F(1, 7943)	= 606.72
				Prob > F	= 0.0000
				R-squared	= 0.0710
				Adj R-squared	= 0.0708
Total	1.1352e+10	7,944	1429021.17	Root MSE	= 1152.3

ofimn	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
osex						
female	-637.3352	25.87467	-24.63	0.000	-688.0563	-586.614
_cons	2062.275	18.64855	110.59	0.000	2025.719	2098.831

Sex and job hours predicting income

```
. reg ofimn ojbhrs i.osex
```

Source	SS	df	MS	Number of obs	=	7,945
				F(2, 7942)	=	794.96
Model	1.8935e+09	2	946761687	Prob > F	=	0.0000
Residual	9.4586e+09	7,942	1190962.07	R-squared	=	0.1668
				Adj R-squared	=	0.1666
Total	1.1352e+10	7,944	1429021.17	Root MSE	=	1091.3

ofimn	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
ojbhrs	33.96065	1.123629	30.22	0.000	31.75804	36.16326
osex						
female	-337.0889	26.44232	-12.75	0.000	-388.9228	-285.255
_cons	787.1759	45.73595	17.21	0.000	697.5214	876.8304

Sex and hours