Quantitative Research Methods: Introduction to correlation and regression

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Multidimensional causality

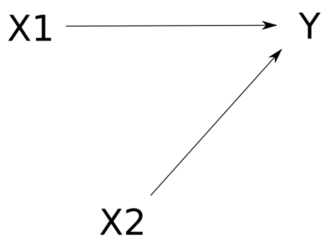
- Regression analysis never proves causal relationships, but it "thinks" in causal terms
- To use it we need to understand causal relationships: what process generates the

data we see, and what can regression tell us about it.

- Start by considering the relationship between variables and patterns of association

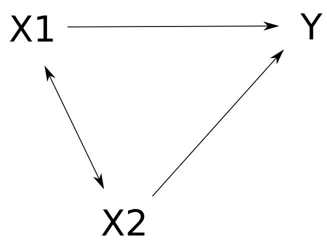
3-variable pictures

- Let's consider patterns of causality and association between three variables, X1 and X2, and Y
- If X1 and X2 are not correlated with each other, their separate effects on Y more or less just add up



Correlated X variables

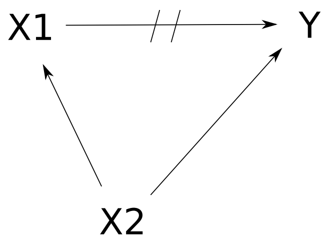
- But if X1 and X2 are correlated, things can get funny:



- In particular, if we measure the effect of one X without taking account of the other we will likely over-estimate it

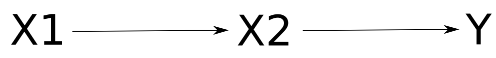
Spurious association

- X1 may have an association with Y, implying a causal relationship
- But if X2 affects both X1 and Y the relationship between X1 and Y may be spurious



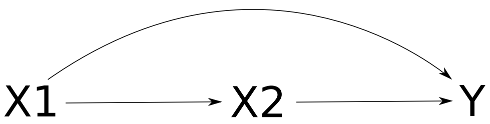
Indirect effects

- Where there is a time-order (X1 before X2), we may see direct and indirect effects
- X1 may affect X2, which affects Y, but not affect Y directly
- Thus there is association between X1 and Y without a direct causal effect



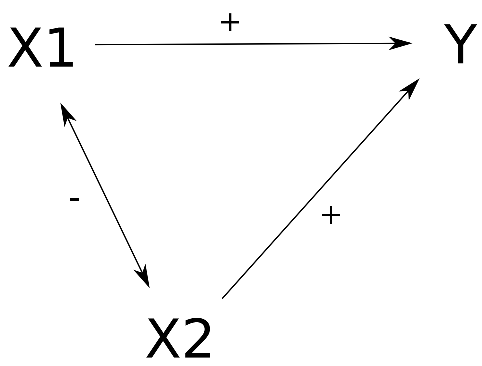
Direct and indirect effects

- However, it is possible for both direct and indirect effects to be present at the same time



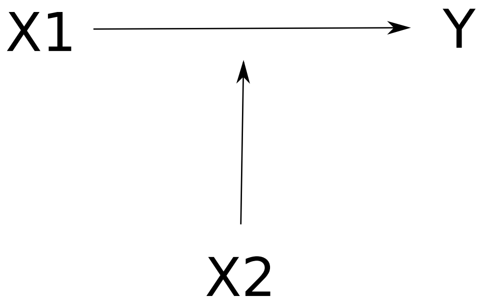
Suppression

- Where X1 and X2 have positive effects on Y, but a negative correlation, or different effects on Y with a positive correlation, the association between X1 and Y may be supressed
- That is, it may be invisible if we don't take account of X2



Interactions

- An interaction effect is where the effect of one variable on Y changes depending on the value of another



Multiple explanatory variables

- Regression analysis can be extended to the case where there is more than one explanatory variable – multivariate regression
- This allows us to estimate the net simultaneous effect of many variables, and thus to begin to disentangle more complex relationships
- Interpretation is relatively easy: each variable gets its own slope coefficient, standard error and significance
- The slope coefficient is the effect on the dependent variable of a 1 unit change in the explanatory variable, *while taking account of the other variables*

Example

- Example: domestic work time may be affected by gender, and also by paid work time: competing explanations – one or the other, or both could have effects
- We can fit bivariate regressions:

$$DWT = a + b \times PaidWork$$

or

$$DWT = a + b \times Female$$

- We can also fit a single multivariate regression

$$DWT = a + b \times PaidWork + c \times Female$$

Dichotomous variables

- We deal with gender in a special way: this is a *binary* or *dichotomous* variable – has two values
- We turn it into a yes/no or 0/1 variable – *e.g.*, female or not
- If we put this in as an explanatory variable a *one-unit change in the explanatory variable* is the difference between being male and female
- Thus the c coefficient we get in the $DWT = a + b \times PaidWork + c \times Female$ regression is the net change in predicted domestic work time for females, once you take account of paid work time.
- The b coefficient is then the net effect of a unit change in paid work time, once you take gender into account.

Income, hours and gender

. corr Income Gender Hours
(obs=506)

	Income	Gender	Hours
Income	1.0000		
Gender	-0.3280	1.0000	
Hours	0.3638	-0.4360	1.0000

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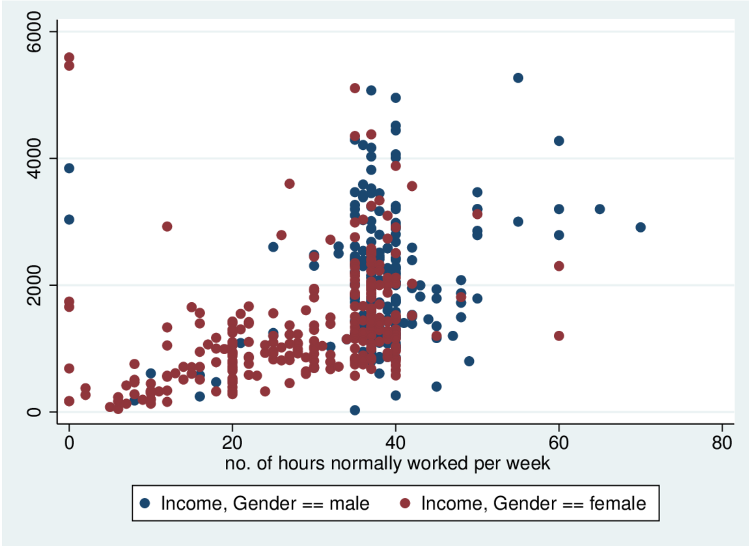
Income, hours and gender



T-test: Income by gender

. ttest Income, by(Gender)

Two-sample t test with equal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
male	437	1618.348	59.11677	1235.809	1502.159	1734.537
female	531	992.1805	40.82127	940.6625	911.9892	1072.372
combined	968	1274.861	36.23219	1127.281	1203.759	1345.964
diff		626.1674	70.00484		488.7883	763.5465

diff = mean(male) - mean(female)   t = 8.9446
Ho: diff = 0   degrees of freedom = 966

Ha: diff < 0   Ha: diff != 0   Ha: diff > 0
Pr(T < t) = 1.0000   Pr(|T| > |t|) = 0.0000   Pr(T > t) = 0.0000

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Regression: Just hours

. reg Income Hours

Source	SS	df	MS
Model	86947928.8	1	86947928.8
Residual	570128215	504	1131206.78
Total	657076144	505	1301140.88

Number of obs	=	506
F(1, 504)	=	76.86
Prob > F	=	0.0000
R-squared	=	0.1323
Adj R-squared	=	0.1306
Root MSE	=	1063.6

Income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
Hours	37.82204	4.314061	8.77	0.000	29.34628	46.2978
_cons	449.7435	150.1722	2.99	0.003	154.703	744.7841

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Regression: Hours and binary gender

```
. reg Income Hours i.Gender
      Source |       SS           df       MS      Number of obs   =       506
-------------+----------------------------------   F(2, 503)       =     50.70
       Model |   110236231         2  55118115.6   Prob > F        =    0.0000
    Residual |   546839912       503  1087156.88   R-squared       =    0.1678
-------------+----------------------------------   Adj R-squared   =    0.1645
       Total |   657076144       505  1301140.88   Root MSE        =    1042.7

------------------------------------------------------------------------------
      Income |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       Hours |   28.33857   4.699451     6.03   0.000      19.1056    37.57155
             |
      Gender |
     female  |  -478.4214   103.3684    -4.63   0.000    -681.5084   -275.3344
       _cons |   1022.139   192.2717     5.32   0.000     644.3844    1399.893
------------------------------------------------------------------------------
```


Regression: for men only

```
. reg Income Hours if Gender==1
      Source |       SS           df       MS      Number of obs   =       232
-------------+----------------------------------   F(1, 230)       =      5.36
       Model |  8009519.02         1  8009519.02   Prob > F        =    0.0215
    Residual |   343845612       230  1494980.92   R-squared       =    0.0228
-------------+----------------------------------   Adj R-squared   =    0.0185
       Total |   351855131       231  1523182.38   Root MSE        =    1222.7

------------------------------------------------------------------------------
      Income |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       Hours |   24.61855   10.63597     2.31   0.022     3.662162    45.57495
       _cons |   1164.366   414.4901     2.81   0.005     347.6826    1981.049
------------------------------------------------------------------------------
```


Regression: for women only

```
. reg Income Hours if Gender==2
      Source |       SS           df       MS      Number of obs   =       274
-------------+----------------------------------   F(1, 272)       =     42.63
       Model |  31772944.2         1  31772944.2   Prob > F        =    0.0000
    Residual |   202744304       272  745383.469   R-squared       =    0.1355
-------------+----------------------------------   Adj R-squared   =    0.1323
       Total |   234517248       273  859037.537   Root MSE        =    863.36

------------------------------------------------------------------------------
      Income |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       Hours |   29.70376   4.549594     6.53   0.000     20.74687    38.66065
       _cons |   504.6153   140.3614     3.60   0.000     228.2824    780.9482
------------------------------------------------------------------------------
```


Regression: interaction

```
. reg Income c.Hours##i.Gender
      Source |       SS           df       MS      Number of obs   =       506
-------------+----------------------------------   F(3, 502)       =     33.82
       Model |   110486228         3  36828742.8   Prob > F        =    0.0000
    Residual |   546589915       502  1088824.53   R-squared       =    0.1681
-------------+----------------------------------   Adj R-squared   =    0.1632
       Total |   657076144       505  1301140.88   Root MSE        =    1043.5

--------------------------------------------------------------------------------
        Income |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------+----------------------------------------------------------------
         Hours |   24.61855   9.076915     2.71   0.007     6.785132    42.45198
               |
        Gender |
       female  |  -659.7502   392.3082    -1.68   0.093    -1430.518    111.0181
               |
Gender#c.Hours |
       female  |   5.085207   10.61255     0.48   0.632    -15.76529     25.9357
               |
         _cons |   1164.366   353.7327     3.29   0.001     469.3865    1859.345
--------------------------------------------------------------------------------
```

Regression: Direct and indirect 1

. reg ownscore fatherscore

Source	SS	df	MS			
Model	13269.3853	1	13269.3853	Number of obs	=	1,000
Residual	247525.861	998	248.021905	F(1, 998)	=	53.50
				Prob > F	=	0.0000
Total	260795.247	999	261.056303	R-squared	=	0.0509
				Adj R-squared	=	0.0499
				Root MSE	=	15.749

ownscore	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
fatherscore	.2370829	.032413	7.31	0.000	.1734773	.3006884
_cons	37.90861	1.672157	22.67	0.000	34.62726	41.18996

Regression: Direct and indirect 2

. reg education fatherscore

Source	SS	df	MS			
Model	311.104929	1	311.104929	Number of obs	=	1,000
Residual	2797.00607	998	2.80261129	F(1, 998)	=	111.01
				Prob > F	=	0.0000
Total	3108.111	999	3.11122222	R-squared	=	0.1001
				Adj R-squared	=	0.0992
				Root MSE	=	1.6741

education	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
fatherscore	.0363018	.0034455	10.54	0.000	.0295405	.0430631
_cons	1.295213	.1777516	7.29	0.000	.9464035	1.644023

Regression: Direct and indirect 3

. reg ownscore education

Source	SS	df	MS			
Model	80742.8091	1	80742.8091	Number of obs	=	1,000
Residual	180052.437	998	180.413264	F(1, 998)	=	447.54
				Prob > F	=	0.0000
Total	260795.247	999	261.056303	R-squared	=	0.3096
				Adj R-squared	=	0.3089
				Root MSE	=	13.432

ownscore	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
education	5.096871	.2409273	21.16	0.000	4.624089	5.569653
_cons	33.87079	.8556481	39.58	0.000	32.19171	35.54986

Regression: Direct and indirect 4

. reg ownscore education fatherscore

Source	SS	df	MS			
Model	81453.7212	2	40726.8606	Number of obs	=	1,000
Residual	179341.525	997	179.881169	F(2, 997)	=	226.41
				Prob > F	=	0.0000
Total	260795.247	999	261.056303	R-squared	=	0.3123
				Adj R-squared	=	0.3109
				Root MSE	=	13.412

ownscore	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
education	4.937369	.2535982	19.47	0.000	4.439722	5.435017
fatherscore	.0578475	.0290984	1.99	0.047	.0007463	.1149486
_cons	31.51367	1.461439	21.56	0.000	28.64582	34.38152