SO5032 Lecture 9

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Outline

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Logistic regression

Outline

Today we introduce logistic regression: for binary outcomes

See Agresti Ch 15 Sec 1.

Binary outcomes and regression

- OLS (linear regression) requires an interval dependent variable
- Binary or “yes/no” dependent variables are not suitable
- Nor are rates, e.g., n successes out of m trials

Problems with OLS

- Errors are distinctly not normal
- While predicted value can be read as a probability, can depart from 0:1 range
- Particular difficulties with multiple explanatory variables
- Nonetheless still often used

Linear Probability Model

- If we use OLS with binary outcomes, it is called “linear probability model”:

$$Pr(Y = 1) = a + bX$$

- data is 0/1, prediction is probability
- Assumptions violated, but if predicted probabilities in range 0.2–0.8, not too bad

Credit card example

```
. reg card income

      Source |       SS           df       MS      Number of obs   =       100
-------------+----------------------------------   F(1, 98)        =     34.38
       Model |  5.55556122         1  5.55556122   Prob > F        =    0.0000
    Residual |  15.8344388        98  .161575906   R-squared       =    0.2597
-------------+----------------------------------   Adj R-squared   =    0.2522
       Total |       21.39        99  .216060606   Root MSE        =    .40197

------------------------------------------------------------------------------
        card |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   .0188458    .003214     5.86   0.000     .0124678    .0252238
       _cons |  -.1594495    .089584    -1.78   0.078    -.3372261     .018327
------------------------------------------------------------------------------
```

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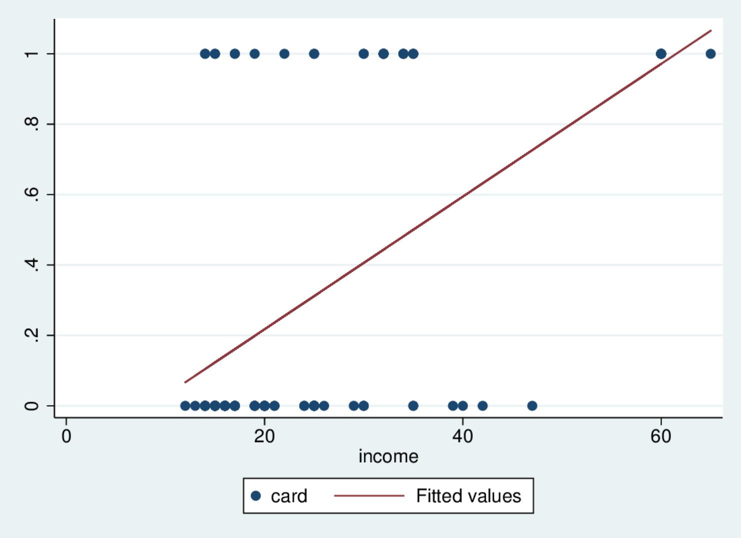
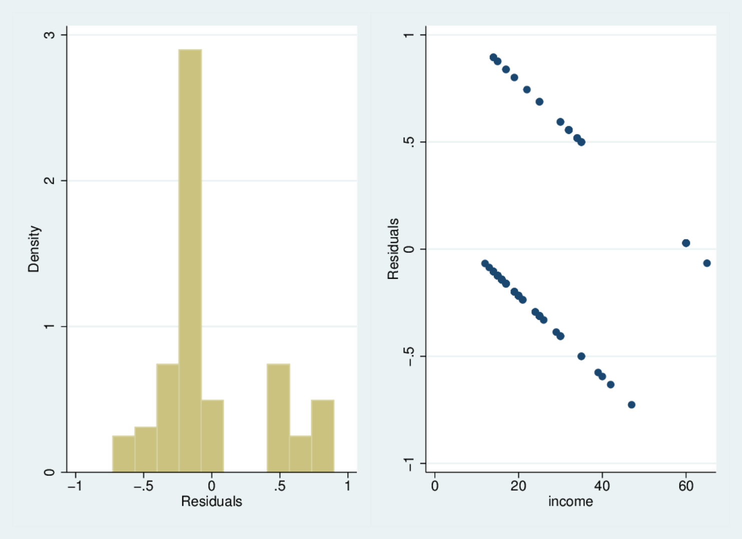
Credit card example



Credit card example



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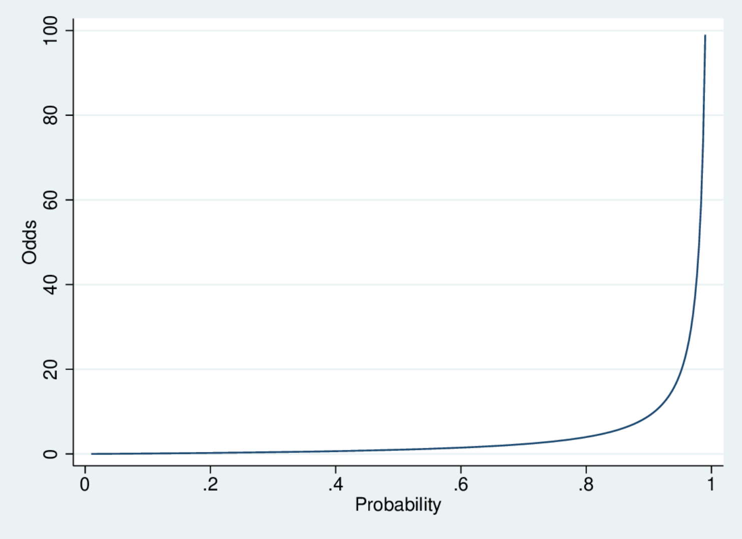
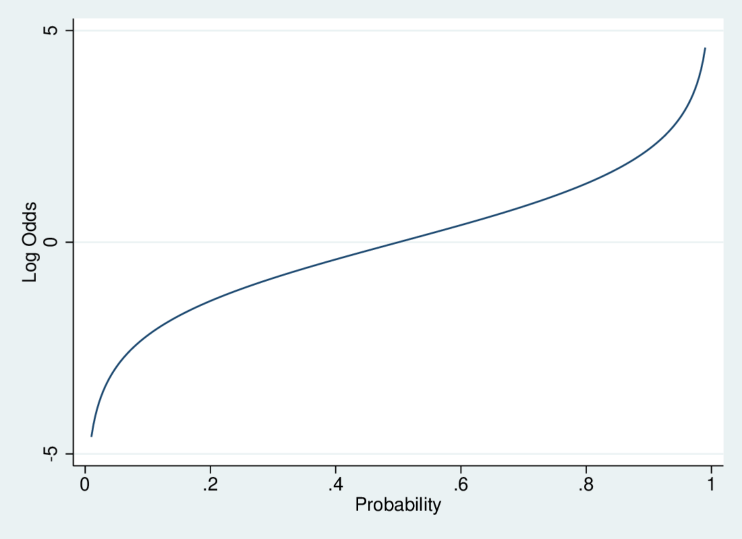
Logistic transformation

- Probability is bounded $[0 : 1]$
- OLS predicted value is unbounded
- How to transform probability to $-\infty : \infty$ range?
- Odds: $\frac{p}{1-p}$ – range is $0 : \infty$
- Log of odds: $\log \frac{p}{1-p}$ has range $-\infty : \infty$

Probability to odds



Probability to log-odds



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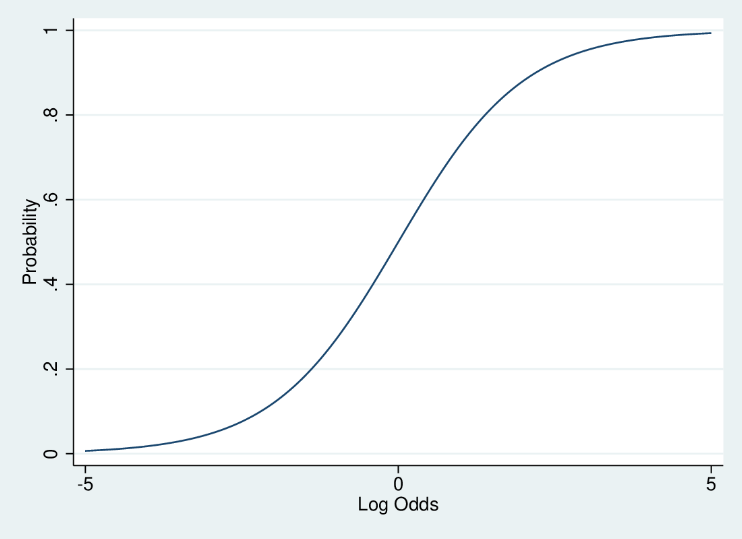
Rotated: the "S-shaped" curve



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Logistic regression

- Logistic regression uses this as the dependent variable:

$$\log\left(\frac{p}{1-p}\right) = a + bX$$

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Alternatives

We can look at this in three ways

- In terms of log-odds:

$$\log\left(\frac{Pr(Y=1)}{1-Pr(Y=1)}\right) = a + bX$$

- In terms of odds:

$$\frac{Pr(Y=1)}{1-Pr(Y=1)} = e^{a+bX}$$

- In terms of probability:

$$Pr(Y=1) = \frac{e^{a+bX}}{1+e^{a+bX}} = \frac{1}{1+e^{-a-bX}}$$

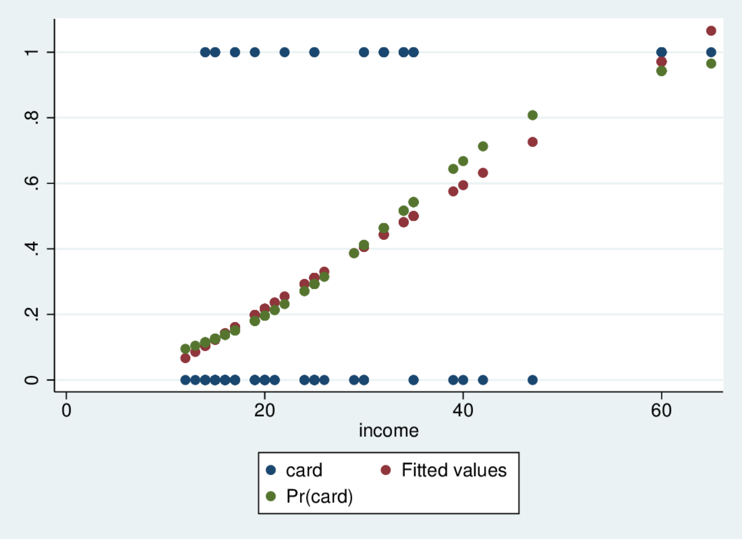
Parameters

- The b parameter is the effect of a unit change in X on $\log\left(\frac{Pr(Y=1)}{1-Pr(Y=1)}\right)$
- This implies a multiplicative change of e^b in $\frac{Pr(Y=1)}{1-Pr(Y=1)}$, in the Odds
- Thus an odds ratio
- But the effect of b on P depends on the level of b

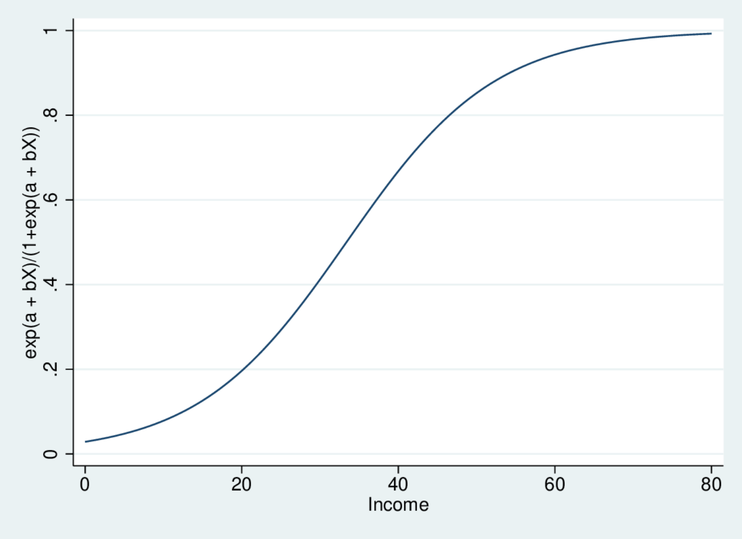
Credit card logistic regression

```
. logit card income

Iteration 0:   log likelihood = -61.910066
Iteration 1:   log likelihood = -48.707265
Iteration 2:   log likelihood = -48.613215
Iteration 3:   log likelihood =  -48.61304
Iteration 4:   log likelihood =  -48.61304

Logistic regression                             Number of obs     =        100
                                                LR chi2(1)        =      26.59
                                                Prob > chi2       =     0.0000
Log likelihood =  -48.61304                     Pseudo R2         =     0.2148

------------------------------------------------------------------------------
        card |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   .1054089   .0261574     4.03   0.000     .0541413    .1566765
       _cons |  -3.517947   .7103358    -4.95   0.000    -4.910179   -2.125714
------------------------------------------------------------------------------
```

Credit card logistic regression



Sigmoid curve from a+bX



Calculating predicted probabilities by hand

- We can calculate the predicted probability for any combination of values of the independent variables
- First, plug them into the a + bX part to get the predicted log-odds
- Then take the anti-log of the log-odds to get the odds
- Then odds/(1+odds) gives us the probability

Calculating predicted probabilities

- Example: log(odds) = 0.25 + 0.12X
- Predict for X == 10
  - Predicted log-odds = 0.25 + 0.12*10 = 1.45
  - Predicted odds = $e^{1.45}$ = 4.263
  - Predicted probability = 4.263/(1 + 4.263) = 0.810

Web applet for practicing

http://teaching.sociology.ul.ie:3838/logabx/